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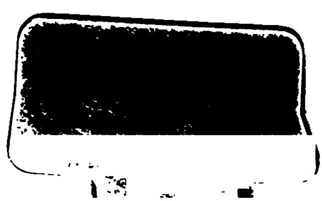

THE GREAT GIANT ARITHMOS

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A MOST ELEMENTARY ARITHMETIC

BY

MARY STEADMAN ALDIS.

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PREFACE.

In the present day the tendency of legislation and custom is in the direction of taking away the sense of parental responsibility in relation to the education of the young. There are still, however, mothers who wish to retain some portion of that influence which nature intended them to have in the training of their children, and who refuse to abandon it wholly either to the schoolmaster or the State. To such as these this little book is offered as a help in laying the foundations of one of the most important branches of instruction. Children are too often expected to work with numbers without understanding their nature, whence arises a state of mental confusion, dislike to the subject, and consequent want of progress, which a careful explanation of first principles would avoid. It will be seen that the book is intended to be read *to* the child. It is divided into lessons, each of which contains as much matter as a child ought to be expected to digest at one sitting, perhaps even more. Should any lesson seem too long to be perfectly apprehended,

it may be divided into two; and in any case the lesson read for the first time one day should be repeated on the following, to see how far it has been really taken in by the pupil's mind. Repetition is the mother of studies—even a little excess of care at the beginning may save much trouble at the end. It is above all urged that the materials mentioned,—the bricks, the pens and writing-book, and coloured ink or pencils,—should be provided and used as from time to time directed. As it is very desirable that the learner should regard the lesson as a matter of importance, it is well that he should be made to feel that the teacher holds the same view. Even the manner of procuring the articles named may be used to strengthen this desirable impression.

One other point must not be forgotten. With very few exceptions, the notes of interrogation denote questions to be answered *by the child.* If the answer should not be forthcoming, it will show that the subject has not been understood, or that it has been partially forgotten. The remedy in either case is to turn back to the earlier page which treats of the question, by which means the answer may always be obtained. Perseverance in this method will be found to sharpen attention and improve the memory.

THE GREAT GIANT ARITHMOS.

DID you ever hear of the great Giant Arithmos?

He is a most wonderful giant indeed, and there are a great many things to be told about him. But there is one thing I cannot tell you, though you may think it strange I should not be able to do so.

When people hear of a giant they are very likely to ask how tall he is. Now if you ask me how tall Giant Arithmos is, I cannot tell you, because I do not know.

Perhaps you will think I might find out; but I never met with any one who knew, and I do not believe there is any one in the world who knows, exactly how tall this great giant may be.

Yet none but a very great giant could do the things which Arithmos can do. I will tell you one of these things. Do you ever look out at night and see the beautiful stars? They shine very brightly though they look so small. Are the stars really very small? No, the stars are

really great worlds, some of them as large as this world we live in, some smaller, and some a great deal larger.

The reason the stars look so small is that they are so very very far away. We can only see these bright worlds as little points of light; but Giant Arithmos can reach the stars—he can, indeed.

Is not he something like a giant? Did you ever hear of one so big?

Giant Arithmos can do many other strange things, but the best of it is he likes to be useful, and is very fond of helping people to do all sorts of things they could not possibly manage without him. He will help grown-up people when they want him, and he is always ready to help children too.

Some people are very fond of the good giant, and cannot think what they should do without him.

There are others who say that they have no doubt Arithmos can be very good to persons whom he likes, but that, for their part, they cannot understand him, and he is of very little use to them.

There are even people who hardly ever speak of Arithmos without calling him some unkind name. They say he is horrid, a cross old thing, and that they hate him, and wish such a giant had never been made.

The reason of all this is that the great giant is

fond of riddles, and when any one wants his help, he says, "Very well, but you must find out some of my riddles." If they want him to do a hard thing, they have to find out hard riddles. If it be an easy thing, the riddles are easy too.

One good thing is that when any one has found out some of the easy riddles, they help to find out the harder ones.

Now many kind fathers and mothers think that it would be very good for their children to be able to get Arithmos to help them whenever they like, so, of course, the children must learn what? Why, to find out these riddles.

This is called learning Arithmetic.

Some children do not like to learn Arithmetic, and so they are always finding fault with the giant, which does not seem quite fair.

But sometimes the reason why children do not like these riddles is, that they have never been taught exactly what they mean, and so it is very hard to find them out.

I am going to tell you about some of the easiest riddles, and to try to show you how to find them out, so that Giant Arithmos may be your very good friend, and then I am sure you will like him very much, and never call him any hard names.

So now you are going to learn Arithmetic, so that Giant Arithmos may be willing to help you when you want him.

Arithmetic is a long word, but even little people need not be frightened at it, for they can understand a great deal about it if they try.

Of course, nothing can be done without trying. Even babies have to try to learn to walk, and you, who are so much older than a baby, can do what a baby does, and a good deal more besides. Don't you think so?

Well then, you are going to learn what? Arithmetic. That is, you are going to learn about numbers, and what can be done with them.

But before you learn what can be done with numbers, there is something you will want to know. Can you tell what that is?

You know what a knife is, and what you can do with it; you can cut things. Perhaps sometimes you have cut your own fingers, but that was a mistake, no doubt. But if I were to give you a little penknife and ask you to cut down a tree with it, or if I gave you a paper-knife and asked you to cut up an apple, what should you say?

You would say that it was not the right kind of knife.

Before you can tell what you can do with a knife then, what do you want to know?

You want to know what kind of a knife it is, you say.

Just the same with numbers. Before you

can tell what you can do with a number you want to know? What kind of a number it is.

Yes, for there are different kinds of numbers as well as of knives, and you must try to make friends with these different kinds of numbers.

When you see a number you must say—"Well, Master Number, who are you, what kind of a number are you, and what can you do?"

Then, when you have got the answers from a good many numbers, you will begin to know something about—about what, do you think?

About Arithmetic.

Now before we begin to ask the numbers the questions I spoke of, we shall want certain things to help us, for I think it will be much easier for you if we do not trust altogether to your counting-machine.

Did you say you had not a counting-machine? Then you must be a very unfortunate child, for every one who is not very unfortunate has a counting-machine.

We shall see; but now I will tell you what else we shall want.

First then, we must have a table all to ourselves, and the table must have nothing on it, it must be quite clear, ready for us to learn arithmetic.

Then, if you have a box of bricks, that will help us very much; if you have no bricks, perhaps you have dominoes or some other toys of

that kind; or if you have nothing at all of that sort, we must have a few pebbles or pieces of stick.

Now, as I cannot know what each child will have, I shall always *say* bricks—and you will know that I mean whatever you choose to use.

We shall want something else too, because you must learn to write the figures.

Some children use slates, but I think it will be much better for you to learn to write like a grown-up person at once, with pen and ink, and the best way will be for you to have a nice book with lines ruled in it, and then the figures will not get rubbed out every day as they would on a slate.

How many things we want! Can you tell them all?

A table, bricks, pen, ink, and book. All to help us to learn what? Arithmetic.

LESSON I.

HERE is the table ready for our lesson, and here are the bricks, but they may stay in their box just now.

Tell me how many bricks are on the table?

Not one.

Then, do you think there is any number on the table that we can begin to question?

No, not one.

You think you might as well put some bricks on the table; but first of all we must have a little talk about Not one. Sometimes in arithmetic we have to talk about Not one, but then we do not say Not one.

We say—Nought.

Now I will write the word for you; not in your book, that is for you to write in, but on a piece of paper, so—Nought.

When you begin to do sums, you will not want to take the trouble to write all those letters every time you have to put down a Nought. People who do sums manage in another way.

Now I am going to ask you a question, which you are to answer without speaking a word, so shut your mouth up tight, and do not open it again just yet.

Here is the question—Do you know what a *sign* is?

If you do not know, I expect you will shake your head; if you do know, you will nod. I shall understand you quite well, and you will have told something without speaking.

That is, you will have made a *sign*.

Let us see if we know any other signs.

When any of our friends want to come to see us, would not it be funny if they were to stand outside the house and call to us, and say—I have come to see you; I want to come in; please open the door?

You can tell what sign they make to let us know they want the door opened. They knock at the door or they ring the bell, and then we know some one wants to come in.

Have you ever noticed the tall posts near railway stations, with the great arms which go up and down? What are they for?

Why, to make *signs* to the engine-drivers, and tell them whether it is safe for the train to go on, or whether there is another train in the way. If there were no signs the train would have to be stopped for some one to come and say whether it might go on. That would be very troublesome, and waste a great deal of time. It is much better to have signs.

There are other signs on the sea-coast at night, very bright ones, which tell the sailors which way to go, so that their ships may not get on the rocks. What are those signs called? Lighthouses. If you ever go in a ship you will be very glad when you see the lighthouse which tells that you are near home.

I daresay you will be able to think of a good many other signs. We will only talk of one other kind — the sign-posts which are set up where two or three roads meet. Can you tell what they are for? To show people which road to go.

We shall find some signs like these in arithmetic, signs to show us which road to go; but

now we will go back to Nought, or Not one, and see how we are to write it more shortly.

The signs for numbers are called figures.

Here is the figure for Nought—0. I will write it for you on the paper.

Is not that much more quickly written than the whole word? Signs generally save time.

Now suppose I told you to take up Nought and bring it to me, could you do it?

Of course not; you cannot take up nothing you say; there must be something if you are to bring it to me.

So nothing can be done with Nought, only sometimes, as I told you before, we want to say that there is Nought.

So we must have a figure for it as I have shown you.

Before our next lesson I want you to write an exercise in your book. Write very carefully and slowly five Noughts, one for each finger on your hand and one for the thumb.

Just fancy if any one asks what you have been learning to-day, you can say Nought, which means nothing; and yet I hope you have learned something. Is not that funny?

LESSON II.

So you have written your exercise, and have put the table and bricks all ready for another lesson.

Then you may take a brick out of the box and put it on the table.

How many bricks are on the table now? One brick.

Why did you put one brick on the table? Because I told you to do so?

Did I tell you to put *one* brick on the table? No, I said *a* brick.

Then why did you put one brick; think, and tell me.

Yes, you did quite rightly. No one talks about a bricks, a tables, a boxes, or a anything that means more than one; it would be nonsense.

Really it almost seems as if *a* meant *one*. And so it does.

You know that sometimes we say *an* instead of *a*—an apple, an egg. *A* apple, *a* egg, would sound very strange.

A long time ago people used to say an or ane before all sorts of words; they would say ane book, ane child, which you see is almost like saying one book, one child.

Then by degrees they got into the way of saying *a* instead of ane before a good many words, and so ane lost sometimes one letter and was called *an*, and sometimes two letters and was called *a*.

So, as I said, you were quite right to put one brick on the table when I told you to put *a* brick there, for it really meant just the same thing.

Before we make the figure for One, I must tell you that there are two kinds of figures.

One kind is called Arabic, and is supposed to have been made a long time ago in a country called Arabia.

Let us get the atlas and find Arabia. See, here it is.

Now let us find England. It is quite a long way off, you see.

The Arabic figures are used now by English people when they want to write figures in their letters or bills, and also in doing sums such as you are going to do. They are shorter, and therefore much more convenient than the other kind of figures, which are called Roman figures.

You think, perhaps, that as the Arabic figures are so much used, and are also more convenient than the Roman figures, it is not worth while to learn the Roman figures.

Ah, but the Roman figures are used sometimes, so that we must learn them.

Besides, some of the Roman figures have a way of almost telling you what they mean, because they would not do at all for any number except the one they stand for.

That is not so with the Arabic figures; each one of them, except the first, would do just as well for any other number, if every one were to agree to call it by the name of that other

number, just as *e* might be called *a* if every one agreed to say *a* whenever they saw an *e*.

Where did the Romans live, can you tell? In Rome.

Let us find Rome on the map. Do you know where it is?

At least you can find Italy; that country in the shape of a boot, you know, at the south of Europe. That is it; and there is Rome.

At first, a long long time ago, Rome was only a small place. Then it became larger and larger, and more people lived in it.

Sometimes the Romans quarrelled with the people who lived in the other towns near to them. Then they fought battles, and very often the Romans gained the victory, for they were very brave.

So little by little they became very strong, and made themselves masters of all Italy, and of many other countries besides.

They came to England, too, and conquered part of it, and built houses, and made statues, and other things here.

Sometimes when people are digging deep into the earth in England, they find pieces of old Roman houses, and sometimes they find money and other things made by the Romans.

If there are any figures on these things, what kind of figures are they; Arabic or Roman? Why, Roman of course.

There were plenty of schools and teachers in Rome, so that the children could learn to read, and could learn arithmetic too.

The Romans used to tell fine tales about what they called the old times of their city. I should think the Roman children were pleased when any one told them these tales. I daresay you would like to hear them too.

Perhaps you will hear them some day. I hope you will, but now we must remember that we have to learn arithmetic, and that we must learn two kinds of figures.

One kind called what? Roman.

And the other? Arabic.

Here is your exercise. Write in your book, very nicely of course, Roman. Write the word five times.

Then write Arabic. Write that five times too.

LESSON III.

DID you ever wonder who made all the figures; whether one man made them all, and if he did, whether he made them all at once, or one or two at a time?

Can you tell anything about it? No.

Neither can I, for I do not know. No one knows.

So, if we like, we may guess, though we cannot tell whether we guess rightly.

I should *think* very likely the Roman figures at least were not made all at once, but a few at a time, as people found they wanted them; just as a baby does not learn all words at once, but one or two at a time as it wants them.

When the Romans were so busy at first building their city, and fighting with other cities, they would not have much time to think about numbers.

When people are not very civilised—that is, have not learned to think very much—they sometimes have not even any words for numbers, so of course they have no figures.

The Romans were not very civilised at first. Afterwards they became so, and when they learned to think about numbers, I think they would find they wanted figures to write these numbers.

At first perhaps they would only learn to think about a few numbers.

Then they would only want a few figures.

Afterwards they would learn to think about a great many numbers.

Then what would they want? A great many figures.

That is my guess, you know; if you think it is not a good guess, you must try to make another better one.

Now I want to have a game. Will you play with me, and will you lend me your box of bricks to play with?

I am going to pretend that the box of bricks is a basket of peaches, and that I am a Frenchwoman who wants to sell the peaches. I cannot speak English, but only French.

Now will you pretend that you are in France, and that you want to buy a peach—only one peach, just to try whether they are good, and because you do not want to spend much of your money?

Oh dear, you have no money to play with; what will you have, buttons, or beads, or pieces of paper? Anything will do to make believe with.

Now we are ready to begin our game.

Will you pretend you are going for a walk in a French town, and that you see I have some peaches to sell?

Come and look at them and ask how much they cost.

Cannot you speak French? No. And the Frenchwoman cannot speak English.

What is to be done? Can you tell?

Do you remember we found out a way of telling things without speaking? If you have forgotten we must turn back and see how it was managed (p. 7).

Ah, now you know. We must make signs.

The Frenchwoman thinks you want to buy as many peaches as you can carry, so she takes up five or six and offers them to you.

You shake your head. She understands that sign very well. So she puts the peaches back into the basket and points to them, and holds up two fingers, so, to see if you want two peaches.

You shake your head again, but now you know how to make the Frenchwoman understand. How will you do it? She held up two fingers; you hold up how many, to show that you only want one peach?

Why, one finger, of course.

Now the Frenchwoman will pick out the best peach in her basket for you. She is so pleased that you have understood each other by signs.

Do not forget to pay for the fruit, if you please. Where is the money? Are you going to give her only one little piece of money?

See what she is doing. Holding up two fingers. What does that mean?

It means that she wants two of those little pieces of money to pay for that nice peach. There, that is right.

But what *would* you and the Frenchwoman have done without your counting-machines.

Do you know now what those counting-machines are?

I think you must have found out that your two hands, each with its row of fingers, make a very good counting-machine.

Let us make another guess about those old Romans. Don't you think that before they had

any names for numbers, they most likely used their counting-machines to make each other understand?

And don't you think that when a Roman began to think what sort of a figure he should

Making Signs.

make for the number One, he would say to himself that he might as well make a mark something the shape of his finger? A long straight mark, you know.

If my guess is right, the Romans were not

quite satisfied with a straight line so—|—but after a time they put two other little lines, one at the top and one at the bottom, so—I—that is a Roman figure One.

What was the other kind of figures called? If you have forgotten you can look in your writing book. Arabic.

This is the Arabic figure One—1—very nearly like the Roman figure.

Now we will rule a line down the middle of a page of your writing book.

Then on one side of the line you can write Roman, and underneath that word you must write I five times.

What shall you put on the other side of the line? Arabic.

That is right, and underneath Arabic you must write 1 five times.

LESSON IV.

Do you remember when we began to learn arithmetic, I said we must ask the numbers questions about themselves to find out what they could do?

Let us try to find out what number One can do.

Put a brick upon the table. Can you bring it to me if I ask you? Yes, you can.

You could not bring Nought, but you can bring One, that is quite a different thing.

Then bring the brick to me.

Why is the brick not upon the table?

Because I told you to bring it to me, and it is in my hand.

But why is it not upon the table at the same time as it is in my hand?

Why you say, the brick could not be in my hand and on the table at the same time.

But why not? Try to tell me.

Because it is only one thing, and one thing cannot be in two places at the same time.

No, it is impossible. One brick may be first in one place and then in another place, but it can only be in one place at one time.

Now put the brick in the box again.

How many bricks are on the table now? Not one.

Here is a pencil and a piece of paper. Show me how you write the figure which means Not one.

You have not forgotten? No, there it is—0.

Put one brick on the table.

What did you do then? You put one brick where there was not one before.

So you may say you put One to Nought, for there was not one till you put the one there.

When you had put One to Nought how many bricks were there on the table? One.

Then when you put One to Nought how many does that make? One.

There! you have done a sum. It is a very

little sum, but it is a real sum which you have done in your little head.

Another day you must learn how to write it. To-day I want you to learn something else.

We said that you put One to Nought. So you did.

But when people do arithmetic they do not talk about putting one number to another, they use another word which means just the same thing.

They say they *add* one number to another.

Add is not a long word, and it means just the same as to put to, for it means to give or join to, and when we give or join one thing to another, we put it to the other thing.

I think you will be able to remember quite easily, that when you put One to Nought you—what did you do? Added One to Nought.

There are a great many different kinds of sums.

When you added One to Nought what kind of a sum did you do, do you think?

An *add* sum? Not exactly.

When you brought your exercise to me did I say the *write* was pretty good? No, that would sound very strange. I said the *writing*, not the write.

But we do not talk about the adding; we say, not adding, but Addition.

So you did an Addition sum.

What does that mean? That you did a sum in which a number was added to what was there before.

In your sum what was there before was Nought. Presently you must do sums in which several numbers are added together.

Now, what does Add mean? To give, or join to. And what does Addition mean? Addition means the giving or joining to.

Here is your exercise. Write Add five times and Addition five times.

And try to remember what they mean, because I shall be sure to ask, and I hope you will be able to tell.

LESSON V.

I HOPE you remember what *add* means, because if not, we must read the last lesson over again.

You know we have to write your sum in Addition to-day. Before we write anything we must understand what we write, or we shall make mistakes.

Now tell me how many bricks there are on the table. Not one.

Take one brick and put it on the table.

What did you do then? You know I hope that you added One to Nought.

And when you had added One to Nought how many did that make, how many bricks were on the table? One.

So that One added to Nought makes One, or we may say it is the same as One.

Let us write that in figures, so—1 added to 0 makes 1, or is the same as 1.

If we have to write so much every time we add one number to another, what a long time our sums will take if there are many numbers in them.

Perhaps we can find some shorter way. Try if you can find out how to manage it.

You have not forgotten about the railway signal and the lighthouse, have you?

And how did you manage to make the French woman understand?

Now you know what we must do. We must have a—a what? Yes, a sign.

Make the sign or figure for Nought on a piece of paper, so—0.

Now I will show you how to make a sign for what comes next in our sum.

What is that? Added to—because you added One to Nought.

Here is a nice little sign +.

Whenever we see that little sign after a number, we know that the number is to be made more by having another number added to it.

Let us write our sum once more in the old way, and then we will write it in the new way, with the sign.

1 added to 0 is the same as 1

0 + 1 is the same as 1

So that sign is called the sign of Addition.

What do the sign-posts by the roads tell people? They tell them which road to go.

So whenever we see this sign + we know that we are to go along the Addition road.

Addition Road.

That is, we know that one number has to be added to another number.

You remember the lighthouses, which are for signs to the sailors (p. 8). They are all called lighthouses, but each lighthouse has a name of its own.

So all the signs used in arithmetic are called signs, but each sign has—tell me what each sign has like the lighthouses? A name of its own.

Then of course this sign + has a name of its own, and of course you want to know that name.

The name is—Plus.

Plus is a Latin word. Latin is a language which is not spoken in any country now; but do you know what people used to speak it?

Our old friends the Romans spoke Latin. The Roman girls and boys a long time ago talked Latin, just as you talk English.

Add is a Latin word too, and means—but *you* can tell *me* what add means (p. 21).

So when we learn arithmetic we have not only to learn how to make Roman figures, but also to learn a word or two of the Roman language which is called Latin.

Plus means more.

The sign + is called Plus which means? Yes, which means more, because whenever that sign is put after a number it shows that that number is to be made more.

How must the number be made more? By having another number added to it.

Plus means more. Suppose I said you were to be plus a piece of cake. What would that mean? That you were to have a piece of cake added to you. Should you like that plus?

To-day you have been plus a lesson. The

lesson has been quite long enough, too, so we will not make it plus another lesson for fear you should be plus a sleepy fit, which would not do at all.

Plus a Slice of Cake.

We must not forget, though, that your writing-book must be + another exercise.

Write the sign and its name five times, so—+ plus.

LESSON VI.

You have put our one brick on the table so often, that I really think you can tell me to-day

without the help of the brick, how many we have if we add 1 to 0. $0+1$ is as much as? As 1.

We made our sum very much shorter yesterday by writing + instead of added to.

Now, if we could find another sign for *makes*, or *is just as much as*, how very much more quickly we should write the sum.

Then we should use only signs, and not have to write any words in the sum.

Well, here is the sign which is used instead of *makes*, or *is just as much as*, =.

You can make that sign, I am sure. Just try on a piece of paper.

Now, all lighthouses are called lighthouses, but each lighthouse has—what? Its own name.

So all signs are called? Signs. But each sign has? Its own name.

What is the name of the sign of Addition? I am sure you know; but if you should happen to have forgotten you must look in your writing book.

What language is plus? English? No, Latin, which the Romans used to speak.

This sign = however, has an English name. Its name is equals—e-quals. You can say that, can you not? Yes, e-quals.

Look at the sign = again. How many lines are there in it? Two lines.

Is one line longer than the other? No, both

lines are just the same length. We may say the lines are *equal* in length.

When we say that two numbers added together are equal to another number, we mean that when the two numbers have been added together they make just as much as the other number.

When we add One to Nought, how many does that make? It makes One. So One added to Nought is just as much as One. It is not any more, and it is not any less. It is just One.

So we say that $0+1=1$.

I think you can guess what your exercise is to be to-day.

Write five times, = equals.

LESSON VII.

Let us write our sum once more in words. Then we will write it with figures and signs and see which looks best.

One added to Nought equals One.

$$0+1=1.$$

You see how much trouble it saves to use signs. It is worth while to take a little trouble to understand them.

$0+1$ means Nought with One added to it.

Would it mean the same if we said $1+0$?

How much would 1 be if you added 0—that is, if you added nothing to it? Just One.

So it makes no difference whether we put 1 first, or 0 first. $0+1$ is just the same as $1+0$.

We may say $0+1=1+0$, for they both $=1$.

How did we manage to make our sum look so much shorter? By using signs.

Suppose now we were to leave out the signs, and just add the figures together without the sign of Addition, would not that be shorter still?

People do often write sums without signs, but then they do not write the figures on the same line, as we have been doing. They put one figure over another like this—

$$\begin{array}{c} 1 \\ 0 \end{array}$$

Then they draw a line underneath so—

$$\begin{array}{c} 1 \\ 0 \\ \hline \end{array}$$

and underneath that they write what it all equals.

What does 0 added to 1 equal? 1. Then we must write it so—

$$\begin{array}{c} 1 \\ 0 \\ \hline 1 \end{array}$$

You know it does not matter which figure we put first, so let us try in how many different ways we can write our sum.

$1+0=1$. That is one way.

$0+1=1$. That is two ways.

$$\begin{array}{c} 1 \\ 0 \\ \hline 1 \end{array}$$ That is three ways.

$$\begin{array}{r} 0 \\ \underline{1} \\ 1 \end{array}$$ That is four ways.

Suppose we were to write a great many Noughts under the One, would that make the One any more?

What does Nought mean? Not one.

Then if we said Not one a great many times, would that ever make One? No, never.

If we had only One and a great many Noughts, the right answer to our sum would still be One.

Before we have another lesson, I want you to write your sum in your book.

How many ways can you write it in? Four ways.

Then write your sum in your book in four ways.

Perhaps you would like to write it on a piece of paper first, so as to be quite sure you have made no mistake. If you do that, you can copy it afterwards into your book.

LESSON VIII.

PUT a brick on the table.

Now there is one brick on the table.

Take the brick away again.

How many bricks are on the table now? Not one.

How is it that there is not one brick upon the table? There *was* one which you put there.

Yes, you say, but I told you to take it away again. Oh, so I did.

Now will you take another brick off the table.

What, cannot you do it? How is that?

Because there is not one there, and when there is not one you cannot take one away.

No, of course you cannot take One from Nought.

But when there was one brick on the table then you *could* take one away. When you had taken *one from one* how many were left? Not one.

Now you have done a new sum. Let us write it—1+. Stop; will that do? Did you add something to One? No, you took One away from One.

Then if you did not add, was it an addition sum? Why no; how could it be an addition sum without adding?

So you see you have not only done a new sum, but you have done a new kind of sum.

In this new kind of sum you did not add One; what did you do? You took One away.

In arithmetic we do not say take away, we say Subtract.

Let me hear you say the word. Sub-tract.

Then instead of adding, what did you do? You sub-tracted.

Subtract is a Latin word which means to take

away from. See how much those old Romans have to do with our arithmetic.

Now I wonder if you can find out what a sum is called when we subtract. You know that when we add it is called Addi-tion.

When we subtract it is called Sub-trac-what do you think? Subtraction.

So you have done some little sums in Addition. And you have done one little sum in Subtraction. You are getting on with the great giant's riddles.

Exercise.—Write Subtract five times.

Then write Subtraction five times.

LESSON IX.

We thought it would be very troublesome to write *add* every time we wanted to add one number to another.

It would be more troublesome to write subtract over and over again. Subtract is a much longer word than add.

I expect you are quite in a hurry to say what must be done, without waiting to be told. Oh yes; of course we must have a sign.

It will not be the sign of Addition; what will it be the sign of? The sign of Subtraction. Here is the sign of Subtraction, –

Now you know three signs. Make them all

upon a piece of paper, and tell me their names, those which you know. Which is the smallest of all? The sign of Subtraction.

We cannot go on always speaking of the sign of Subtraction. What must the sign have quite of its own, like the other signs? It must have a name.

Well then, the name of the sign of Subtraction is Minus. Minus is a Latin name like Plus.

I really think that if you try you will be able to find out the meaning of Minus for yourself. I will help you a little.

What does Plus mean? You hardly *can* have forgotten, I suppose; but if you happen to have a very bad memory we must just read the lesson on Plus over again (p. 21).

Plus, then, means *more*. When it is put after a number what do we know is to be done to that number? Another number is to be added to it, and so it will be made *more*.

When we see Minus—the sign of Subtraction —after a number, does it tell us that *that* number is to be made more? No, it tells us that something is to be taken away or subtracted; so the number will be made—not more—but? But *less*.

Then Minus does not mean more. It means what? It means less. I thought you could find out that Minus means less.

Exercise.—Write the sign - and the name Minus five times.

Minus the Cake.

LESSON X.

PUT a brick on the table. Now take a piece of paper and write the number of the bricks on the table. How many is that? 1.

Take the brick away; how many are left? Not one.

Then if you subtract One from One, what does that equal? It equals Nought.

Let us write it with the signs. $1 - 1 = 0$. One minus One equals Nought.

You see the first 1 has the sign of Subtraction after it, and that shows it is to be made not more but? Less.

And so it *was* made less, for when 1 was subtracted it equalled only 0.

That poor brick has been moved about so much, do you think it is tired? Shall we put it on the table and leave it there now for a little while?

Then how many shall we take away? None at all. Then if we take none away, how many will be left? Just One.

So if we take Nought from One, One is left.

Let us write that properly. $1 -$

What does that mean? That we are going to subtract something from One.

But we made up our minds to subtract nothing this time, and we are going to say so. $1 - 0$. That is, One made less by Nought. That we know equals One. $1 - 0 = 1$.

Do you remember that when we used the sign of Addition, we found that it did not matter at all which figure we put first? (p. 27) $1 + 0 = 0 + 1$. For $1 + 0 = 1$, and $0 + 1 = 1$.

Do you think it is the same with the sign of Subtraction? Can we put which figure we like first? Let us try. $1 - 0 = 1$.

Now let us put it the other way, $0 - 1$. What does that mean?

That One is to be subtracted from Nought; that is, that Nought is to be made less by having One taken from it. Can you do that? No, when we began to learn about Subtraction we found out that you could not take One from nothing (p. 30).

So in Subtraction sums we must be careful to put the right figure first, because it is quite plain that we cannot take a number from another number which is smaller than itself.

Signs help us to do our sums more shortly than words. What way did we find of doing them more shortly still? (p. 28) You can show in your book the shorter way of doing Addition sums.

Now for Subtraction. First we write the number which is to be made less so—1.

Then underneath we write the number which is to be subtracted so—
$$\begin{array}{r}1\\1\end{array}$$

Then we draw a line, just as in Addition, and write how much it makes underneath—
$$\begin{array}{r}1\\1\\\hline 0\end{array}$$

We will do our other Subtraction sum as well—
$$\begin{array}{r}1\\0\\\hline 1\end{array}$$

You see we cannot write our Subtraction sum in as many ways as we wrote the Addition sum (p. 28). How is that?

Because in Addition you can put either of the figures first, but in Subtraction you cannot do so. You have to be careful to put the right figure first.

Do you know how many Subtraction sums we have done? We did two, and now we will write them in both ways, with the signs and without them. $1 - 1 = 0$, $\begin{array}{r}1\\ 1\\ \hline 0\end{array}$ $1 - 0 = 1$, $\begin{array}{r}1\\ 0\\ \hline 1\end{array}$

Write these sums in your book. Then write your Addition sum again in all the four ways. Then you will have all your sums together.

LESSON XI.

WE found that we could do our sums more quickly by using signs than by using words. Then we found a quicker way still. What was that? We left out the signs.

Can you think of a shorter way still? Suppose we were to leave out the figures. Why, then we should have nothing to write! No, and we cannot always do sums in that way. Sometimes it is much better to write them. But when we want to do a sum in a hurry it is a very good thing to be able to do it without writing, to do it in our heads, as it is called.

So now let us *say* our sums. I will say the numbers and you shall say what they equal. Now then! $0+1=?$ $1-0=?$ $1-1=?$ $1+0=?$

Now put a brick on the table. That is one brick. Really we are getting tired of seeing that one brick all alone on the table. To-day it shall have a companion.

Put another brick on the table. What does *an*other mean? What does *an* mean? Shall we turn back to Lesson II. and see? (p. 10). Then we shall find that *an*other means *one* other. How many bricks are on the table now? Two. $1+1=?$ Two.

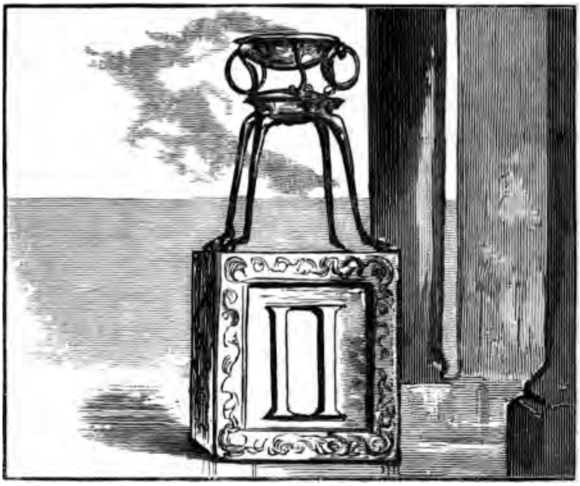
You can tell, I hope, in how many different ways numbers can be written, how many sorts of figures there are? And you know what are the names of those different sorts of figures? At any rate, I am not going to tell you. That would not be worth while when you have written them in your book and can find them there.

If we were right in our guess that some old Roman thought he would make a mark something like one of his fingers when he wanted to write One, what do you think he would do when he wanted to write Two?

If he made a mark like one finger for One what would he make for Two? I think he would make two marks. At all events, we know that the Roman figure Two is made so. This is it—II.

If you ever see any of the altars or tomb-

stones which the Romans left behind in England so long ago, look carefully and you will very likely find a Two, such as I have shown you, cut in the stone.



Roman Altar.

When you knew that I. stood for One do you not think that you would almost have known without telling that II. stood for Two?

Here is the Arabic Two—2, which you could not know without being told.

Exercise.—Write Roman on one side of your book, and write Arabic on the other as you did before when you learned to write One.

Then write the Roman Two under Roman,

and the Arabic Two under Arabic, each five times. When you have finished write on the line underneath I. II. 1, 2.

LESSON XII.

BEFORE we go on with this lesson let us say over again the same sums as we said yesterday, so as to be quite sure of knowing them well.

Now let us do a sum in Addition. Write Nought on a piece of paper. What kind of sum are we going to do? Then you know what sign to make, +. Now write 1. Now you have $0+1$.

We have done that before, and we know what it equals. It equals? 1.

But to-day we are going to add something else, so what must you write next to show that there is more addition to be done? You must of course write the sign +.

Now we will add another 1. Write 1 in its place after +. Read to me what you have written. $0+1+1$.

Now we must have the sign for equals—$0+1+1=$. What does it equal? I believe you can tell without the bricks, but you may put them on the table if you like. First one brick, then another brick. How many bricks altogether? Two bricks, $0+1+1=2$.

When we had only one brick on the table how many more had you to add to make two?

Only one more, was it? Then Two is how many more than One? Two is One more than One.

Put the bricks in the box again. Now how many are on the table? None at all.

Is your hand large enough to take two bricks out of the box at once, instead of taking first one and then another? Put them both on the table.

How many did you add then? You added Two. $0+2=?$ $0+2=2$.

Two more than Nought is just Two, just as One more than Nought is just One.

$$1+1=2$$
$$0+2=2, \text{ so}$$

$1+1=0+2$, because each of those little sums equals Two.

So we have learned that One added to One, or $1+1$, equals? Two.

And we have learned that Two is One more than One.

And we have learned that $1+1$ is just as much as $0+2$.

Here are some sums which you may do, and write in your book. We will write them in both ways.

$2+0=$. What? *You* must put the answer. $0+2=?$ $1+1+0=?$

0	2	1	0
1	0	0	2
1	—	1	—
—		—	

LESSON XIII.

LET us say some sums. $0+1$? $0+2$? $1+1$? $0+1+1$?

To-day we will do some sums in Subtraction. Put two bricks on the table. Write Two.

What sums are we going to do? Then what sign must come next? Minus. What does minus mean? Minus means less (p. 32).

Two is going to be made less by having something subtracted from it. Take one of the bricks off the table. How many are left? One.

Then $2-1=$? 1. So you can finish the sum.

How many less than Two is One? One is One less than Two.

Take away one more brick. None are left.

We will write another sum with another minus in it. $2-1-1=$? What does it equal when you have subtracted first one and then the other?

It equals Nought, for nothing is left.

$$2-1-1=0.$$

Put the two bricks on the table again. There they are. Now take them both away at once. Not first one and then the other. How many did you take away? Two. And none are left.

Then Two subtracted from Two equals Nought.

$2-2=0.$ $\quad 2-1=1$

$\quad\quad 0+1=1$, so

$2-1=0+1$, because both $2-1$ and $0+1=1$.

That is, $2 - 1$ is just as much as $0 + 1$.

When there is one brick on the table how many more must you put to make two? One more.

Why must you put one more? Because if you want to have Two you must have two Ones. It takes two Ones to make one Two.

Put two bricks on the table; that is, two Ones. Now take all your bricks out of the box, and put them in Twos all over the table. Every Two has how many Ones? Two Ones.

Do you know what we call two Ones when we speak of things?

What have you on your feet? Shoes. How many shoes, one? No, a pair. And how many is a pair? A pair is two. You have known that ever since you were a baby, before you could speak. If only one of your shoes had been put on, you would have held out your foot for the other. You knew very well that you ought to have two shoes, if you never thought about a pair being Two.

So you have put your bricks in pairs.

When we speak of two things of the same sort, we say a pair.

When we speak of numbers and mean two Ones, we say a Two, or one Two. Sometimes we speak of a great many Twos.

We have learned to-day that $2 - 1 = ?$ 1.

And that $2 - 1$ is just as much as $0 + 1$.

And that $2 - 2 = 0$.

Can you tell me what $2 - 0 =$?

Two, if you subtract nothing from it, is still Two, so $2 - 0 = 2$.

A Pair of Shoes.

We must not forget the sums for your book. Subtraction sums to-day.

$2 - 1 = ?$ $2 - 0 = ?$ $2 - 2 = ?$ $1 - 1 = ?$ $1 - 0 = ?$

2	1	2	1	2
1	1	2	0	0
—	—	—	—	—

LESSON XIV.

Let us first look at the sums in your book. Then we will *say* them.

Put one brick upon the table.

÷

"We cannot both have it."

Suppose we both wanted that brick, could we both have it at the same time? No, that would not be possible. What should we do then—should we begin to cry? We would not be such babies, would we? Then should we quarrel

about it? Why, we could not think of being so silly as that. If we cried all day, or quarrelled all day, that would not make it possible for us both to have the same one brick at the same time. But what *should* we do? I think I should say that though I wished for the brick you should have it because you are the younger, and then, I think you would say I should have it because I am the elder. If we did that we should have learned a little piece of a very difficult lesson, a lesson which is more difficult and more important than arithmetic even. Do you know what lesson that is? It is the lesson of *being willing to give up our own way.* Grown-up people have to learn that lesson as well as children, but those learn it most easily who begin when they are children.

Now put another brick on the table.

How many are there now? Two. Two what? Each brick is one brick, so when there are two there are two Ones. When you put your bricks on the table in pairs, how many Ones were there in each pair? Two Ones.

Then could you have One and I have One if we wished to do so? Yes; Two is quite different from One, because it *can* be divided between two people.

Suppose I were to tell you that some one meant to give us a present, that the present was two nice things of some sort, but that I had not

been told what these two things were, only that they were to be divided between us. You would understand and be able to think about the number Two, though you would not know two *what*.

And you could tell how many you would have, and how many I should have. How many would that be? We should have One each. How do you know that? Because you know how many Ones there are in Two. How many are there? There are two Ones in Two.

Now you have done another sum. Think; was it an Addition sum when you told me how many Ones there are in Two? It was not Addition, for you did not add.

Was it a Subtraction sum? It was not Subtraction, for you did not subtract. You only told how many of one number there are in another number. It must be a new kind of sum. So it is.

This new kind of sum is called Division. Di-vi-sion. It is called Division because it teaches how to divide a number into equal shares.

You divided Two into Ones. You knew how many times *One* would *go* into *Two*, as it is sometimes called.

Of course Division must have a sign as well as Addition and Subtraction.

Here is the sign of Division, ÷ a line with one dot above and one beneath. That sign ÷ has not

a short Latin name like + and –. It is used to show that one number is to be divided by another number, so when we see it we say, Divided by.

Now you know the names of three kinds of sums. Let me hear you say them—Addition, Subtraction, Division.

What does + mean after a number? (p. 24). That that number is to be made more by having another number added to it.

What does – mean after a number? (p. 32). That that number is to be made less by having another number subtracted from it.

And what does ÷ mean after a number? That that number is to be divided by another number.

For your exercise write Divide and Division each five times.

Also write ÷ Divided by, five times.

LESSON XV.

$2 \div 1$. What does that mean?

It means that 2 is to be divided by 1; that is, 2 is to be divided by the number which comes after the sign.

You know how many Ones there are in Two, so we can do that sum. $2 \div 1 = ?$ 2.

Now let us make an Addition sum which shall come to Two. I believe you can do it all by yourself. Then you will have made a sum and

done it all yourself. Have you managed it? $1+1=2$. So $1+1=2\div1$, because both $=2$.

$0+2$? $2-0$? These two sums also $=2\div1$.

Yesterday you put a great many Twos of bricks on the table. To-day I want one Two. There it is.

How many Twos is it? One Two.

$2\div2$.

Then how many Twos are there in Two? One Two. Let us write it. $2\div2=\mathbf{1}$.

How many Ones are there in One? In one One there must be just? One One. We will write that also. $1\div1=$? **1**.

Are there any Twos in 0? How could there be when Nought means—what does Nought mean? Not one. $0 \div 2 = 0$.

$1 \div 1 = 1$. What other Division sum have we done which gives an answer 1? $2 \div 2 = 1$. So $2 \div 2 = 1 \div 1$, because both $= 1$.

Let us make one Addition sum which comes to 1, and one Subtraction sum.

First the Addition sum. What must we add 1 to, to make it $= 1$? We can only add it to 0, for if we say $1 + 1$ that will be too many. $0 + 1 = 1$.

Now the Subtraction sum, how shall we do it? $1 - 1 = 0$. That will not do, but $2 - 1 = 1$. That *will* do.

So we have done one Addition sum, one Subtraction sum, and two Division sums, which all come to One.

Did you ever hear of Jack and Jill, who went up the hill? I suppose they were two children.

Jack and Jill went up the hill
To fetch a pail of water.

I suppose they had got their pail of water, and were coming down when—

Jack fell down and broke his crown,

Then how many children were left to take care of the pail? Only one, for $2 - 1 =$? 1.

The next thing you know was that

Jill came tumbling after.

So 1 child + 1 child fell down; that is, two children fell down, for $1+1=2$. Do you think they went home − the water, and if they did, do you think they were + a scolding. Poor Jack and Jill! It would be hard to be + a fall apiece and + a scolding as well.

Fly away Jack.

Do you know the rhyme about the other Jack and Jill?

Two little blackbirds sat on a hill,
One named Jack and one named Jill.

That was $1+1$, you see.

Fly away Jack, fly away Jill.

When they had both flown away how many were left? None at all, for $2-2=?$ 0.

Come again Jack, come again Jill.

When Jack and Jill both came back, how many blackbirds were there? Two, for $0+2=?$ 2.

Before we leave off I must show you how to do Division sums without signs. Division can be done without signs as well as Addition and Subtraction.

$2\div 1=2$. Let us do that without signs. Which is the number to be divided? Two is the number to be divided. How do we know that? By the sign of Division after it.

Very well, then, we write 2. What is it to be divided by? Two is to be divided by One. How do we know that? Because One comes after the sign of Division.

Then we will make a nice little corner for the number by which Two is to be divided, and put One in that corner so—1) 2

Then we must draw a line underneath to make a place for the answer—$\begin{array}{r}1)\,\underline{\;2\;}\\ 2\;\end{array}$

Let us do another sum in that way—$\begin{array}{r}2)\,\underline{\;2\;}\\ 1\;\end{array}$

And, $\begin{array}{r}1)\,\underline{\;1\;}\\ 1\;\end{array}$

Now I want you to make some sums yourself and write them in your book. Make two Addition sums, two Subtraction sums, and two Division sums. Then if you are not tired, you may make some more of any sort you like. Don't forget to write the answers.

How grand it will be to make the sums your own self, instead of having them made for you.

LESSON XVI.

I WONDER how many sums you have made. Perhaps we had better say a few before we go on. $2-1$. $1+1$. $2\div 1$. $2\div 2$. $1-1$. $2-0$. $0+2$. $1\div 1$.

Put two bricks on the table. Now another. How many are there now? Three bricks.

When there were only Two, how many did you add to make Three? You added One more.

Then how many more than Two is Three? Three is One more than Two.

How many more than Nought is Three? You can tell that directly. Three is Three more than Nought.

Three is Three more than Nought, and Three is One more than Two.

How many more than One is Three? Will Three be as many more than One as it is more than Nought? No. Why not? Because One is One more than Nought, so it will not take so

many to make Three, as if we had not that One to begin with.

How many less will it take to make Three, than if we had not that One? It will take One less.

So as Three is Three more than Nought, how many more than One is Three. Three is Two more than One.

Take one of your bricks to the other side of the table. Set it by itself. How many are left? Two are left.

If you take those two and set them by the one that was sent away all by itself, you will have three all standing together again.

So if you have One, how many must you add to make Three? You must add Two to One to make Three.

Lay one of your bricks flat on its side, as if it had fallen down. Lay another by its side, and another by the side of that. How many bricks have you now? Three.

Now put one more brick on the middle one of the three, and two more on one of the outside ones.

There you have three steps, do you see? The first has only one brick. The second has? Two. And the third? Three. We may call the table *Nought*. Then the first step is *One*. The second? *Two*. And the third? *Three*. Each step is one brick higher than the one before.

We shall soon become very good friends with Three. You know we have to ask the numbers questions and find out what they can do, as much as we can. We must try to find out all their secrets, and not let them hide things from us, so that we may find out Giant Arithmos' riddles. You *have* found out some things about Master Three, so now we will tell a little tale. I daresay you have heard it before. If you have you can tell it for me.

Three blind mice, three blind mice,
See how they run, see how they run;
They all ran after the farmer's wife,
And she chopped off their tails with a carving
knife.
Did ever you see such a thing in your life
As three blind mice, three blind mice?

What a cruel woman! How many were there of the poor little tails which she cut off? Three tails, of course, because there were three mice, and mice only have one tail apiece, have they?

Suppose one mouse was white and the others gray, how many white tails were there, and how many gray?

There would be one white tail belonging to the white mouse, and two gray ones belonging to the two gray mice. You know that.

Now suppose there was only one gray mouse,

and the others were white. How many white tails would there be, and how many gray? Then there would be two white tails and one gray one. Poor little mice.

Poor Blind Mice.

Now I think we had better learn how to make the figures for Three, and then we shall be able to do some more sums.

Show me how to tell the number without speaking, with your counting-machine, you know. You hold up three fingers? That is right.

How would the Roman be likely to make his figure Three. Take a pencil and show me how you think he would do it. Did you make three strokes? That is right. III. That *is* the Roman Three. It seems to say, "Look at me, my name is Three. My name is not Two; I have one stroke too many; and I have two strokes too many to be called One. I should not like to be mistaken for One, who is a poor creature, hardly worth anything at all, and Two is not much better. Please to take notice that *I* am called Three."

Dear me, what a fuss to make about being a little better off than other people—figures, I mean. The Arabic figure Three will not have nearly so much to say for himself, though he is much more convenient for use than the Roman Three.

This is the Arabic Three—3.

You know how to write Roman and Arabic in your book and put the figures underneath—five of each.

Then on another line write One, Two, Three in Roman figures, and One, Two, Three in Arabic figures.

LESSON XVII.

TELL me how many more is Three than Two? One more. That is right.

Put three bricks upon the table, at one end.

Now put three more at the other end, and three more at one side.

You remember when you put your bricks in pairs? (p. 42) Did you put as many bricks together then as you have put now? No. What was the difference? Then you put two bricks together, now you have put three.

What did I tell you we call a pair when we speak of *numbers*, not of things? (p. 42) I think you know. How many are there in a pair? Two.

Then in speaking of numbers we say a Two, or two Twos, or three Twos, or as many Twos as we wish to talk about.

Now you can tell me what we have on the table. They are not Twos, are they? No; they are Threes. There are three Ones in each set.

How many Threes are there on the table? Three Threes. You might set up all your bricks in Threes, but we will not stop for that just now.

Put one Three in the box. How many Threes are left? Two Threes. $3 - 1 = ?$ 2.

Now put another Three in the box. How many Threes are left? One. $3 - 2 = ?$ 1.

Take the one Three that is left and make it into Twos. Have you done it? How many Twos are there? Only one Two, and one brick besides.

What shall we call that one brick to show that it will not make a Two?

How many shoes are there in a pair? Two shoes.

Suppose when you got up in the morning you found that some one had put three shoes by your little bed, what should you say? You would say,

"Why, there are *three* shoes!"

"Oh! here is an *odd* shoe." If there were one more, there would be two pairs or two Twos; but an odd shoe by itself is of no use.

So when we find a number which cannot be made into Twos without having One over, we may

call the One over an *odd* One, and we call the number an *odd* number, because there is an odd One in it. Only, odd numbers are not like odd shoes, for odd numbers *are* of some use.

Let us divide those three bricks between us. Can we divide them equally ?

Do you know what to divide equally between you and me means ? It means to divide so that you shall have just as many as I have, and I shall have just as many as you have.

Now give me two bricks. How many are left for you ? Only One.

Then we have not divided equally. I will give you one of my bricks. Now how many have you ? You have Two, but I have only One. We have not divided equally yet.

Can we divide three bricks equally between us ? No ; we cannot do it. It is not possible to divide Three into two equal parts.

What do we call a number which cannot be divided into two equal parts ? We call it an odd number.

What do we call a number which *can* be divided into two equal parts ? We call a number which *can* be divided into two equal parts, an even number.

I think you know what is meant by two equal parts ; can you tell ? It means two parts of which one is just as much as the other.

What kind of a number is Three, then—odd or

even? Three must be an odd number. Why? Because we cannot possibly divide Three into two equal parts. One part must be One more than the other.

Is number Two odd or even? Why, of course, number Two is even. You can have One and I can have One, so number Two can be divided into two equal parts, and is an even number.

How about number One? Why, who ever heard of such a thing as to call One a pair or a Two! You remember Roman figure Three called One a poor creature. One is so poor that he has not even enough to make one Two. So poor One is an odd One all by himself.

Did you ever play at Odd and Even? Let us have a game. Give me the box of bricks. Hide your eyes. Now I have some bricks in my hand; guess whether I have an odd number or an even number in my hand. Open your eyes and see whether your guess is right. If you have guessed rightly you must take the bricks and try to puzzle me. If you have not guessed rightly I must try to puzzle you again.

What exercise must you write? Even and odd? Yes; I think so, five times each.

LESSON XVIII.

WHAT kind of numbers are 1 and 3? 1 and 3 are odd numbers.

And what kind of number is 2? 2 is an even number.

$2+1=?$ 3. Now let us put the 1 first and say $1+2$. What does that equal? $1+2$ we know is just as much as $2+1$. + is the sign of? Addition. Yes, and in Addition we may put which figure first we like (pp. 27, 34).

Hold up two fingers and then one. That is just the same number as if you hold up first one finger and then two.

When you say $1+2$ and then say $2+1$ you keep just the same numbers; you do not subtract or divide anything.

$3+0=?$ 3. Now we will put it the other way. Write it yourself. Let me see; $0+3=3$. That is right.

I am going to write some Addition sums, but I shall leave you to put the answers, for you can do it quite well.

1	2	1
0	0	0
2	1	1
—	—	1
		—

Put two bricks on the table. Add another. Now you have? Three. Take away the one

you added. Now of course you have just as many as you had before you added the one. $3 - 1 = 2$, because you have to add One to Two to make Three.

$3 - 2 = ?$ 1. That is, you have taken away or subtracted 2 from 3. You may know that the answer is right, because if you added again the 2 you have taken away, you would have 3 again; $3 - 2 = 1$.

Now add the Two again to the One you have left, $1 + 2 = 3$. You may do this with your bricks if you like.

$3 - 0$? If you subtract Nought from Three you must still have Three left. $3 - 0 = 3$. And $0 + 3 = ?$ 3. For 3 is Three more than Nought.

$3 - 3 = ?$ 0. If you have Three and then subtract them all, or take them all away, there will be nothing left at all. $3 - 3 = 0$. And $0 + 3 = ?$ 3.

Subtraction sums.	3	2	3	2	3	3
	2	1	0	2	3	1
	—	—	—	—	—	—

Have we three bricks on the table? We had better have them there. How many Threes are there in Three? In one Three there is just one Three, as you can see. $3 \div 3 = 1$.

How many Ones are there in Three? Three Ones. $3 \div 1 = 3$.

How many Twos are there in Three? You

remember when you tried to make Three into Twos how many there were; but you can try again if you like (p. 57).

There is one Two in Three and One besides, or One over, as it is called.

It is best to write this sum in the other way which I showed you before (p. 51). 2) 3

1

But if we leave it so we shall have said nothing about the One over, and that will not do. We must tell about that One over.

This is how it must be written—2) 3

1—1

That little line is almost like the sign of Subtraction, but it does not mean minus, because you see we are not doing the sum with signs. That little line is only put to make a place for the One that is over. 1) 3 3) 3 1) 2 2) 2

Are there any Noughts in Three? How could we make a number out of Noughts, that is out of nothing? We cannot make anything out of nothing, not even one thing, so of course we cannot make three Ones out of nothing.

I think you can make some nice sums for yourself, and you can write them in your book. How many kinds of sums can you make? Addition, Subtraction, Division, that is three kinds.

LESSON XIX.

PUT three bricks on the table. Now another. How many are there now? Now there are four. So $3 + 1 = ?$ Four.

Let us turn the figures round. You know we may do that in Addition (pp. 27, 34, 61). $1 + 3 = ?$ Four. That is right.

We have not talked about number Four yet. What question must we ask first? Master Four, are you an even or an odd number? What does Four say, do you think? Can he be divided into two equal parts? Yes, he can. So Four is an even number.

How could we tell that Four is an even number? We could tell that in several ways.

If we look at your four bricks, we can see directly that we can divide them equally between us. How many should we each have? You would have? Two. And I should have? Two.

We know that Three is an odd number. There is an odd One in it. How many did we add to Three to make Four? We added One to Three to make Four.

Then the One which we added, and the odd One in Three would make how many? The two Ones would make Two. And Two is an even number, as you know.

So when we add One to an odd number what kind of a number do we make? One added to an odd number always makes an even number, because the One we add and the odd One in the odd number make Two, and Two is an even number.

Tell me all the even numbers we have talked about. Four and Two are even numbers. So they are; but let us say the smaller one first—Two, Four.

Now tell me the two odd numbers. One, Three. That is right. One, Three. Two, Four.

Suppose you went to a shop to buy two pairs of gloves. When you opened the parcel at home how many gloves should you expect to find? If you had bought two pairs of gloves you would expect to find four gloves in the parcel, should you not? If you only found three gloves, what should you say? I think you would say—"Why, the shopkeeper has made a mistake, he has only given me one pair of gloves and an odd one. We must go back and ask him for the other, or else this odd glove will be of no use." You would want an even number of gloves, and you would know that you wanted four gloves if you had bought two pairs.

Hold up four fingers. How many more fingers have you on that hand? No more fingers, only a thumb.

One, two, three, four fingers on each hand,

1 little finger + 3 larger ones. Or 3 larger fingers + 1 little one.

Take a pencil and show me how you think the Roman would write Four. Have you made four strokes? Well, the Romans did sometimes write Four in that way, so you were not wrong; but afterwards they found out another way of writing Four.

I think you will understand that other way of writing Four better when you have learned a little more Arithmetic, so I will not show it you just yet.

This is the Arabic Four—4. We shall want the useful Arabic figure to help us to do our sums.

You can write Roman and Arabic in your book, and write five Arabic Fours, but leave a place for the Roman Fours till I show you how to make them.

LESSON XX.

TAKE one brick from the box. Put it at one end of the table.

Then take two bricks and put them at one side of the table.

Now put three bricks at the other end.

Last of all, put four bricks at the other side.

What numbers of bricks are there on the

table now? No; I do not want you to count all the bricks and tell me what *number* there is altogether. I want you to say how many there are in each little heap or company.

Here at one end is a One all by itself. I do not see another One all alone, so there is one One.

Then there is one Two, one Three, and one Four. Four sets of numbers all different.

Do you see that the odd numbers—which are they? One and Three. Yes; the odd numbers One and Three—are opposite each other? What are the other two numbers? Two and Four—the even numbers, and they are opposite each other also.

One even number comes between two odd numbers, and one odd number comes between two even numbers.

Put One and Three in the box. Look at Two. How many must you add to Two to make it equal to Four on the other side of the table? One would not be enough, you know that. No, $2+1=?$ 3.

Two must be added to Two to make Four.

Then how many Twos are there in Four? You can see that there are two Twos in Four.

How many Twos make Four? tell me again. Two Twos make Four.

When we talk of taking one number twice, like two Twos; or when we talk of taking it

many times, as three Twos, or four Twos, or any number of Twos, do you know what kind of a sum it is?

It is not just like Addition. In Addition we add one number to another; we do not generally add the same number over and over again, though we might do so if we liked; but in this new kind of sum it is always the same number which is taken more than once.

Is it Subtraction? Do we subtract anything? No; we do not subtract anything when we take the same number over and over again.

Then is it Division? In Division we divide a number into parts and so make it smaller.

When we take a number over again we do not make it smaller; we make it? Larger. Yes; two Twos are more than one Two, that is quite plain.

Then our new sum is neither? Neither Addition, Subtraction, nor Division. This kind of sum has rather a long name. Its name is Mul-ti-pli-ca-tion.

The name Multiplication comes partly from a Latin word which means *many.*

When you do Addition do you subtract? No, when you do Addition you add.

So when you do Multiplication you *multiply.* That is, you take one number more than once. Sometimes when you multiply you take a num-

ber a great many times, sometimes you do not take a number very many times.

How many times must we take Two to make Four? Take Two out of the box. Now do it again? Now you have Four. How many times did you take Two? One Two was not enough, you had to take another. You took Two twice.

So twice Two is? Twice Two is Four.

Do you know what Twice means. It means two times. It is easier to say twice than two times, especially when children are in a hurry to finish their lesson.

If twice means two times, what does twice One mean? Twice One means One taken two times. I think you know quite well what twice means, so we need not say two times any more.

Take twice one bricks off the table, and put them in the box.

How many bricks did you put in the box, one? One is only one One, and you took twice One, which is the same as? Twice One is Two.

And twice Two? Twice Two is Four. Put twice two bricks on the table.

How many kinds of sums have you learned about now, do you know? You have learned the names of twice Two kinds of sums. Can you say their names?

Addition, Subtraction, Division, and Multiplication. Four kinds of riddles.

Write in your book Multiplication and Multiply—five times each.

LESSON XXI.

I FEEL sure you can tell what we have to learn to-day. Multiplication must have a sign as well as the other kinds of sums. Here is the sign of Multiplication, $\times$.

Make it yourself on a piece of paper.

This sign has not a Latin name any more than the last sign we learned about. When we see the sign of Division, we say Divided by, and when we see the sign of Multiplication, we say Multiplied by.

2×2. What does that mean? Two multiplied by Two. That is, Two must be taken twice. How many does twice Two make? $2 \times 2 = 4$.

1×2? Twice One is Two. $1 \times 2 = 2$.

Now tell me what $1 \times 3 = ?$ One taken three times equals Three; so $1 \times 3 = 3$.

And $1 \times 4 = ?$ 4. We say that $1 \times 4 = 4$. I think you understand that. You know that if you take one brick out of your box, you have one brick which you can put on the table, and that if you do the same thing four times—that is, if you take four Ones—you have four bricks. So $1 \times 4 = 4$.

Now what does 4×1 mean? It means that you are to take one Four all at once. If you take one Four how many have you? Why, just Four. So $4 \times 1 = 4$, just the same as 1×4.

You see that in Multiplication it does not matter which figure stands first. May we put which figure we like first in all kinds of sums? No, if we did so, some of our sums would be wrong.

Tell me in which kinds of sums must we be careful to put the right figure first (pp. 27, 34). If you have forgotten, we must turn back to the old lessons, but I hope you remember that in Subtraction and Division we have to see that the right figure comes first. In Addition, you know, as well as in Multiplication, it does not matter which figure is put first.

How many kinds of sums have you learned about; can you tell? Say their names. Addition, Subtraction, Division, Multiplication. Four kinds of sums.

Now it is quite time we left off talking about *kinds of sums.* Instead of saying that, we must say *Rules of Arithmetic.* These four kinds of sums (I must use the words just this once) are called the first four Rules of Arithmetic.

Do you know what a Rule is? A Rule is a sort of law. Sometimes a Rule is a law which tells us *when* to do things. Sometimes a Rule tells us *how* to do things.

Do you know any Rules? Is it not a Rule for you to wash your hands before dinner? That Rule tells you *when* to do something. And, of course, you make a Rule for yourself to wash your hands very clean, and so you have to make a Rule to use soap when you wash your hands. That Rule of yours tells you *how* to do something.

I daresay you never thought that you made Rules for yourself; you did it without thinking. Perhaps if you think you will find that you have made some more Rules. As you grow older, very likely you will make other new Rules. I hope they will be very good ones, and that you will be able to keep them. That is sometimes a hard thing to do. So you must try very hard to keep your good Rules.

What do the Rules of Arithmetic tell us? They tell us *how* to do something.

What does Addition tell us? Addition tells us how to Add. And Subtraction tells us? How to Subtract. And Division? How to Divide. And our new rule, Multiplication? How to Multiply.

These four rules are like four brothers and sisters who are alike in some things and different in others. Miss Addition and Miss Multiplication may put whichever figure first they please, but Master Subtraction and Master Division must be careful to put the right figure first.

Then these four rules help each other, as brothers and sisters should. If Addition is not quite sure whether her sum is right she can ask Subtraction to come and subtract the number she has just added; and when he has done that, if

A Happy Family.

the number left is the same as it was before she added, that shows that the sum is right. $1+3=?$ 4.

Now subtract the number added; what is that? Three. $4-3=?$ 1.

If there are some bricks already on the table,

and you put some more, and then take them away again, just as many bricks will be left on the table as there were before you put the new ones there. Try it.

Sometimes Subtraction gets Addition to help him by adding the number which he has just subtracted. $4-3=?$. 1. $1+3=?$ 4.

If there are some bricks on the table, and you take some of them away, and then add them again, there will be just as many as there were before you took any away.

So if Multiplication is puzzled, Division may sometimes say—Oh, I know how many of a smaller number there are in a larger number, and of course you must take just so many of the smaller number to make the larger number. Division knows there are two Twos in Four, so of course Multiplication must take two Twos to make Four.

Or Multiplication knows she has to take two Twos to make Four, and that helps Division to find out how many Twos there are in Four, which is his business.

Write in your book, $\times$ = multiplied by, five times.

LESSON XXII.

TO-DAY we must really do some sums. We have had so much to talk about that we have done very few lately.

$4 \div 2 = 2.$ $\quad 3\overline{)\,4}$

How many Threes are there in Four, do you know? There is only one Three in Four and one One besides. One over, as it is called.

$$\begin{array}{r} 3\underline{)\,4} \\ 1\text{—}1 \end{array} \qquad \begin{array}{r} 4\underline{)\,4} \\ 1 \end{array}$$

That is a very easy sum.

$$\begin{array}{r} 1\underline{)\,4} \\ 4 \end{array}$$ And that is easy too.

Now here is an Addition sum—

$$\begin{array}{r} 1 \\ 0 \\ 2 \\ \underline{1} \end{array}$$

Quite a long sum; but it is not at all too hard for you, is it?

Here is another—

$$\begin{array}{r} 3 \\ 1 \\ \underline{0} \end{array}$$

And one more—

$$\begin{array}{r} 0 \\ 1 \\ \underline{3} \end{array}$$

Now for some Subtraction,

$$\begin{array}{r} 4 \\ \underline{3} \end{array} \quad \begin{array}{r} 4 \\ \underline{1} \end{array} \quad \begin{array}{r} 4 \\ \underline{2} \end{array}$$

You can write them with signs if you like—

$4 - 1 = ?$ $4 - 3 = ?$ $4 - 2 = ?$

Hold up your hand. How many fingers have you? Four fingers. And what else? One thumb. So if you count the thumb as a sort of finger (and it *is* like one, only it is set on one side by itself) how many does that make? One, two, three, four? Five.

Is Five an even or an odd number? You remember what an even number is, or shall we turn back and see? (p. 59).

What kind of number is Four? When you look at your two Twos of fingers, you see that Four is an even number, because it can be divided into two equal parts.

Then if you add an odd One to an even number you see that the odd One makes the number odd. So Five is an? An odd number.

So it is like which other numbers that we have learned about? Like One and Three (p. 65).

Point to your thumb and then to your fingers and say—

> One, Two, Three, Four, Five,
> Fish all alive.

Now begin with your little finger and come back, one finger for each fish.

> This is sole, and this is whiting,
> This is salmon, and this is ling;
> How shall we call the one that's odd?
> In this funny game we call it cod.

You know I told you that though the Romans sometimes wrote Four in this way, IIII, they did not generally write it so. Suppose they had written Four always in that way, and Five like this, IIIII.

Do you think that would have been a good plan? If they were in a hurry don't you think they would be very likely to make mistakes? And if they had gone on making a new line for every new number, how very awkward that would have been. Why, the large numbers would have reached all across the page, and further.

Now my guess is that when the Roman we have spoken of came to Five, he thought he might as well have one figure which should stand for this number, which is the number of all the fingers on one hand. Just as he had made a stroke something the shape of a finger to stand for One, so he would make *something*, he did not quite know what, to stand for Five.

But what should it be? One line would not do, because that stood for One, and the new figure was to stand for Five.

Perhaps he looked at his hand to see if that would help him, and perhaps his hand was lying flat on the table by him with the thumb not quite close to the other fingers. Lay your hand in that way on a piece of paper.

Then take a pencil and draw it along by the side of your first finger and your hand till you

[illegible] of the [illegible].

Look at your hand and see what kind of a figure you have made. It will be something like this— [illegible]

Then perhaps the Roman thought he might as well have [illegible] the same [illegible] as he made it like this—V.

That is the Roman Five, and here is the Arabic Five—5.

We must not forget the Roman figure Four. You will be able to understand it now.

When the Roman had made Five as V, he put One before it in this way—IV, and said that Five with One before it should mean One less than Five.

What is One less than Five? Four is One less than Five.

So now you can write Four in the place you left for it in your book.

And you can write V and 5 in the same way as you have written the other figures.

LESSON XXIII

Hold up your hand and keep all the fingers straight. Now bend your little finger with your thumb. You are obliged to bend them. How many fingers are left straight? Three fingers. Then 5 − 2 = ? 3. Three fingers are left unbent.

$5 - 2 = 3$. Suppose we took one more away. how many would be left? If we took one more away, of course we should have one less left, so as $5 - 2 = 3$, $5 - 3 = ?$ 2.

How many more than Four is Five? Five is One more than Four.

Then how many Fours are there in Five? One Four and One over.

Then if there is one Four, how many Twos are there in Five? You know how many Twos there are in Four? (p. 67). There are two Twos in Four, so there must be two Twos in Five, and One over. Look at your hand and see the two Twos of fingers and the one thumb over.

Suppose you add the One to one of the Twos, $2 + 1 = ?$ 3. Then how many have you besides? Just the other 2. So $3 + 2 = ?$ 5. And $2 + 3 = ?$ 5 (p. 27). And $2 + 2 + 1 = ?$ 5. $4 + 1 = ?$ 5. $1 + 4 = ?$ 5.

How many more Twos than Fours are there in Five? There is one Four, and there are two Twos. So there is one more Two.

Yes, or we might say that there are as many again Twos as Fours in Five. Do you know what as many again means?

Take one brick from your box. Put it on the table. Now take again as many as you took before. How many was that? One. Now, how many bricks are upon the table? Two. Then Two is as many again as One.

Two bricks are on the table. Take again as many out of the box. Now there are Four. Four is as many again as Two. You see it is only another way of saying twice as many, and you know what that means (p. 69).

Whenever we speak of two things, we know that that means two Ones.

When we speak of Four, that does not mean two Ones—but? Two Twos.

Suppose we had a great many Fours (you can put some Fours of bricks on the table if you like), how many Twos would there be in each Four? There would be two Twos in each Four.

If there were not, it would not be Four at all, but some other number. It *always* takes two Twos to make one Four.

And Two is as many again as One. So, however many Fours we have, we always have as many again Twos, or twice as many Twos as Fours.

If we have one Four, we have how many Twos? Two Twos.

If we have two Fours, how many Twos shall we have? Two Twos in each Four, $2+2=4$.

So if we have two Fours, we shall have four Twos, and Four is as many again as? As Two.

Now for some sums—

Addition—

1	0	2
2	3	0
0	1	3
2	1	3

Subtraction—

5	5	4	5	5	5
4	3	3	1	0	2

Division—

2) 5   3) 5   1) 5   0) 5   4) 5

Multiplication—$1 \times 5 = ?$

Make some sums yourself, and write them in your book.

LESSON XXIV.

HOLD up your hand and count the five fingers once more. One, Two, Three, Four, Five. Now hold up one of the fingers of the other hand. How many fingers does that make? Six.

Then how many more than Five is Six? Six is One more than Five. $5 + 1 = ?$ 6. Now let us put it the other way (pp. 27, 71). $1 + 5 = ?$ 6.

What kind of number is Six, even or odd?

Six is an even number. It is One more than? One more than Five, which is an odd number. So the odd One in Five, and the One more to make Six, added together make Two, which is an even number.

Let me hear you say all the even numbers we have learned about. Two, Four, Six. Now say them again very quickly. Now backwards, Six, Four, Two.

And now the odd numbers. One, Three, Five. Now again as quickly as you can. And now backwards, Five, Three, One.

Put one brick on the table. Then next to it put two bricks. Leave a little space between. Now you have one One and one Two. Let us have also a Three, a Four, a Five, and a Six, all in a row.

Look at these numbers of bricks, and you will see that every other number is even, and every other number odd. That is, one even number always comes between two odd ones, and one odd number always comes between two even numbers (p. 67).

Because, if you add One to an even number that One makes the even number odd; and then if you add One more to the odd number, that One makes the odd number even. Try it with the bricks.

Do you remember how the Romans wrote Five? Make the figure. V. And Four? IV. That is

right. The Romans put I before V to show that they meant One less than Five.

But when that old Roman we have so often talked about wanted to write Six, he seems to have thought he might as well write V first, and then I after it—*after*, not before it, to show that he meant Five and One more, which is? Six. So now you can write the Roman Six—VI. And this is the Arabic Six—6.

Write these two Sixes in your book, each five times, and do not forget to write Roman and Arabic each over its own set of figures.

Then write One, Two, Three, Four, Five, Six, first in Roman and then in Arabic figures.

But before you do that, I want you to get some red ink and a new pen. Then you must write all the odd numbers with red ink, and all the even numbers with black ink. How pretty your exercise will look, and how grand you will be with two inkstands and two pens.

LESSON XXV.

SAY the even numbers again. Two, Four, Six How many more than Two is Four? Four is Two more than Two.

And how many more than Four is Six? You know that Six is One more than Five, and Five is One more than? Than Four. So Six must be Two more than Four.

First we have one Two, then two Twos, that is Four; then three Twos, that is how many? It must be Six. So it is.

Write it in this way—$2 \times 2 = 4$. $2 \times 3 = 6$.

Now let us change the place of the figures in the last little sum, as we may do in Multiplication (p. 71), and see what they will tell us. $3 \times 2 = 6$.

What is it these figures tell us? Why, that there are two Threes in Six.

Take three Twos of bricks out of your box, and put them on the table in Twos. Now how many bricks are on the table? Six bricks are on the table.

Take a brick from one of the Twos, and put it to one of the other Twos. How many bricks are on the table now? Six.

Yes, there are still six bricks on the table, but not three Twos. What have we instead of the three Twos? We have one One, one Two, and one Three.

Now take the one brick and put it against the Two.

How many bricks are there now? Still six you say, and so there must be, for you have not put any more on the table, nor taken any away. But you have done something with the six bricks. You have arranged them differently.

First you put three Twos on the table, then you moved one brick, and we had? One One,

one Two, and one Three, and now we have? Two Threes.

Let us pretend that those six bricks are six people who are going to have dinner together. The box will do for the table. Now how shall we set their chairs? We might set six chairs all on one side of the table, but that would not be very convenient. We might put three on one side of the table, and how many on the other? Three on the other. Yes, we might do so, or we might set only two on the other side, and one at the end.

But do you not think the best way would be to put one chair at each end of the table, and two at each side. Then we should have three Twos of people, you see. Now when you have set your brick people in their places, lift up the box very carefully, and see what kind of figure they make. Take a pencil and piece of paper and make a pattern of it with little crosses so—

×
× ×
× ×
×

Then try what other kind of figure you can make with six crosses. You will find you can do so,

× × ×
× × ×

or so,

× ×
× ×
× ×

and you will always have three what? Always

three Twos. Or two? Or two Threes in Six.

Now I wonder whether you would like me to tell you a tale, or shall we put it off till to-morrow? I will write it for you at any rate.

Once there was a little girl named Annie; she was eight years old, and she had a brother named Johnnie. How many children were there? One girl and one boy, that is two children. It was Johnnie's birthday, and he was six years old, so his cousin Harry was to come to tea; he was to come very early, so that the children might have plenty of time to play in the garden. It was a very large garden, with many flowers and fruit-trees, a most delightful place. Now Johnnie was in a great hurry for Harry to come, and directly after breakfast he began to think the time long, and to ask his mother how long it would be before his cousin came. Then his mother said she wanted him and Annie to go into the garden with her; and she gave Johnnie a pretty basket to carry, and she went to look at a large apricot tree where many apricots were getting ripe in the sun.

"*How* old are you to-day, Johnnie," said his mother—"five"? "No, mother," said Johnnie; "you know I'm six." "Well," said his mother, "here are six apricots quite ripe;" and she gathered them and put them in the basket which Johnnie was holding. "Now," said she

"when Harry comes, how many will there be of you?" Annie and Johnnie both called out, "When Harry comes there will be"—Tell me how many there would be? Three. Yes, they were two, and one more would make? Three. Then the children's mother said that as Johnnie was king of the day, he should have the apricots to divide equally between himself, his sister, and his cousin, if he could tell how many he must give to each.

Now Johnnie knew a little about Giant Arithmos, so he was able to tell that there are how many Twos in Six? Three. Yes, and so the three children would each have how many apricots? Two apricots each.

So Johnnie took the six apricots into the house very carefully, and Annie gathered some leaves to put them on, and they did look so pretty and smell so nice, that the two children were sure that the third child, that is Harry, *would* be pleased. But then there came a note for their mother, and when she had read it she said that she was so sorry to have to tell them that Harry had a very bad headache, and could not come to play with them. Then the children were very much disappointed of course, and Johnnie wanted to know what he should do with the apricots, now that Harry was not coming to eat his two. His mother said that Johnnie must divide Harry's share between Annie and himself, so he gave one

apricot to Annie and took one himself. Then how many apricots had each of the children? Three. Yes, because there are two Threes in Six.

But just as the children were going to eat their fruit, Annie said, "Oh, Johnnie, don't you know that little girl who is always ill, and who lives in a house where there is no garden? Should not you like to give Harry's share to her? I think she would be so pleased." "Oh yes," said Johnnie, "if mother will let us." His mother said it was a very good thought, and that the children might carry their present to the little girl themselves. So they divided the apricots once more; they took one from each of their plates and carried them to the little girl, who was very pleased, and thanked them very much, and wished Johnnie many happy returns of the day.

How many apricots did they carry to the little girl? Two. And how many did Annie have to eat? Two. And Johnnie? Two. That was three Twos, and three times Two make? Six.

Write in your book $1 \times 2 = 2$
$2 \times 2 = 4$
$3 \times 2 = 6$

And also $1 \times 3 = 3$
$2 \times 3 = 6$

LESSON XXVI.

THE last was rather a long lesson, so we must try to make this a short one. Read what you wrote last in your book. You began by writing $1 \times 2 = 2$.

That is quite right. What does it mean?

It means that if One be taken twice, there will be Two.

So we say Twice One is Two. Twice Two is Four. Twice Three is Six.

What does 1×3 mean? That One is taken how many times? Three times. So we say Three times One is Three, and so on.

Tell me once more how many Threes there are in Six? Two Threes. Then whenever you speak of Six, you speak of two Threes.

If there were less than two Threes, the number could not be Six, but some smaller number.

If there were more than two Threes, the number could not be Six either, but some larger number.

Now tell me, if there are two Threes in one Six, how many Threes are there in two Sixes?

In two Sixes there must be two Twos of Threes, and two Twos make? Four.

So in two Sixes how many Threes? Four Threes in two Sixes. Twice as many Threes as Sixes.

How many Twos are there in one Six? Three Twos in one Six.

Then how many Twos are there in two Sixes? Three Twos in one Six, and three Twos in another Six; and two Threes make Six.

So there must be Six Twos in two Sixes. Three times as many Twos as Sixes.

One Six, how many Twos? Three. Two Sixes, how many Twos? Six.

One Six, how many Threes? Two. Two Sixes, how many Threes? Four.

So if you had a great many Sixes you would know that you had twice as many Threes, and three times as many Twos, as Sixes.

SUMS.

1				
2	3	1	1	2
1	1	2	4	1
2	2	1	1	2

Write all these sums with the sign of addition after you have done them in this way.

$2\overline{)6}$ $2\overline{)5}$ $3\overline{)5}$ $3\overline{)6}$ $3\overline{)4}$ $2\overline{)3}$

5 − 4. 6 − 2. 4 − 3. 6 − 5. 6 − 3. 4 − 3. 6 − 4.

1 × 6. 1 × 4. 2 × 2. 1 × 5. 3 × 2. 2 × 3.

LESSON XXVII.

HOLD up your hand. How many fingers are there, counting the thumb?

Now hold up one finger of the other hand. How many does that make? Six.

Now one more, and that makes Seven.

So Seven is how many more than Six? Seven is One more than Six.

Write that with the sign of addition. This is the Arabic Seven—7. $6 + 1 = 7$.

Now put the second figure first (pp. 27, 71). $1 + 6 = 7$.

If you subtract One from Seven, how many will be left? Six.

Now write that with the sign of Subtraction. $7 - 1 = 6$.

Yes; and $7 - 6 = ?$ 1. So it does; if you take Six from Seven only One is left.

How many more than Five is Seven? That is very easy, because you know that when you had held up one hand you had to hold up—how many fingers of the other hand to make Seven? Two fingers. So Seven is? Two more than Five. Write $5 + 2 = 7$. And $2 + 5 = 7$.

Now I quite think you can tell me how the Romans wrote Seven. You know that to write Six they first made a figure which meant as many as all the fingers on one hand, V, and then

wrote One after it, because Six is One more than Five.

Then how did they write Seven? V with two Ones after it, because Seven is Two more than Five. VII. That is right; I thought you could find out.

In our last lesson but one you remember, we pretended that six persons were dining together. What did we think was the best way to place their chairs? Two at each side, and one at each end. Now suppose, after they sat down, another person came in, number Seven, where should he sit? Take some bricks and try. Number Seven might sit at one end, so—

```
   × ×
 ×     ×
 ×     ×
    ×
```

or he might sit at one side so—

```
    ×
  ×   ×
  ×   ×
  ×
     ×
```

but he would always be an odd one because Seven is? An odd number.

We know it is an odd number because it comes next to Six, which is an even number (p. 64), and can be divided into two—two what? Two Threes.

If we were only making a pattern on paper we might do like this—

×
× ×
×
× ×
×

and put number Seven in the middle; but we

Number Seven.

could not set a real person in the middle of the table to eat his dinner, could we? That would not do at all.

So we find that Seven is an odd number; that it is Two more than Five; and that the Romans wrote it thus—VII. You know how to write the Arabic Seven, so—7.

What coloured ink shall you use to write Seven?

When you have written it five times, write all the numbers you know, the even ones in black, the odd numbers in red ink.

LESSON XXVIII.

Seven is an odd number; you cannot divide it by Two without having something left over.

How many Ones are there in Seven? Seven Ones, of course, you say; you have to take seven Ones to make one Seven.

Now I am going to teach you a new word. In Arithmetic, when we wish to speak of Ones, we often say Units. What does Units mean?

When a Roman spoke of One, he said *Unus*. Units comes from *Unus*, which means One. Units just means Ones; not Twos nor Threes, but Ones.

How many Units are there in Seven? You know quite well that there are Seven Units in Seven.

Now I want you to say the little piece of the Multiplication Table which is at the end of Lesson 25 (p. 88). You should say that every day before

you begin a new lesson. Sometimes I will remind you; sometimes, perhaps, you will remind me.

Tell me all the odd numbers you know. 1, 3, 5 7. Say them quickly, and then say them backwards. 7, 5, 3, 1.

If you have two bricks, how many more must you take to make seven?

If you hold up two fingers on one hand, you know it takes all those on the other hand to make seven; that is, it takes? Five.

Then if you have two bricks you want five more to make seven. $2+5$? 7.

The Roman figure Seven tells you that. VII. Five, and two more strokes.

How many Units are there in Six? In Four? In Two? In Five? In Three? In One?

Write Units = Ones, five times.

LESSON XXIX.

SAY your Multiplication Table.

How many Twos are there in Six? Three.

Then how many Twos are there in Seven? Three, and One Unit over.

Now, can you tell how many Twos there are in two Sevens? Two Sevens, with three Twos in each; that is, $3+3$ Twos. $3+3=$? 6. So there are six Twos in two Sevens. And One over did you say? Ah, but there is One over in each

of the Sevens, so that makes Two over, does it not? Six Twos and two Units over in two Sevens. How many Twos does that make altogether? Seven Twos in two Sevens.

How many Threes are there in Seven? You know how many there are in Six? There are two Threes in Six.

And in Seven, how many? Two Threes and one Unit over in Seven.

Then how many Threes are there in two Sevens? $2 + 2 = ?$ 4. Then there must be four Threes and two Units over in two Sevens. Two Threes and one Unit over for each of the Sevens.

Put three bricks on the table in a row. Then three more in another row opposite. How many are there now? Six.

Now put one more brick between the two rows at one end— ×

× ×
× ×
× ×

How many now? Seven.

Now begin at the end opposite the odd one, and count the bricks—one, two, three, and then four comes to the odd one, and there are three more afterwards, as well as three before.

So Four is the middle number in Seven.

If Seven were not an odd number there would not be a middle number, because each row would be just equal to the other.

How many days are there in the week? Seven.

And which is the middle day of the week? It must be the fourth day; Sunday one, Monday two, Tuesday three — then Wednesday is the fourth day of the week, and so it is the middle day.

On Wednesday three days of the week are gone, and there are three more to come, and then a new week begins, and soon Wednesday comes again, the middle of another week.

If you take Four from Seven, how many will be left? When four days of the week are gone, how many are to come? Three.

$7-4=?$ 3. $3+4=?$ 7. $7-3=?$ 4. $4+3=?$ 7.

SUMS.

		2	5
		0	0
3	4	1	1
2	0	3	0
1	3	1	1

$7-2.$ $7-4.$ $6-2.$ $7-5.$ $5-3.$
$7-3.$ $4-3.$ $5-2.$ $6-4.$

3)7 2)6 2)7 4)7 4)6 6)7 2)5

$4 \div 2.$ $4 \div 1.$ $6 \div 3.$ $7 \div 1.$

LESSON XXX.

Do not forget to say the little piece of Multiplication.

Hold up one hand. Now two fingers of the other hand. $5+2=?$ 7. And $2+5=?$ 7.

Now one more finger, and that makes Eight. $7+1=?$ Eight. Then $1+7=8$. I think you must know quite well by this time, that in Addition we can put whichever figure first we choose (p. 71), so that it does not matter whether we say $7+1$, or $1+7$. Both equal? Eight. How many more than Five is Eight? How many fingers of the second hand did you have to hold up? Three.

Then $5+3=?$ Eight. And, of course, $3+5=?$ Eight.

How did the Romans write Eight? You can tell, I feel sure. VIII.

Yes, certainly, Five, and Three more.

This is the Arabic Eight—8.

Of course you will write them each five times in your book, but what coloured ink will you use? Is Eight odd or even? Even, because it comes next to Seven, which is an odd number (pp. 67, 82).

You remember how hard it was to find a good place for number Seven when he came to dinner with the other Six. But if eight people were to

dine together, how would they sit? How many Twos are there in Eight? $6 \div 2 = ?$ 3. $6 + 2 = ?$ 8. $6 + 1 = ?$ 7. $7 + 1 = ?$ 8. So $6 + 2 = ?$ 8. That makes another Two.

How many Twos are there then in Eight? Four Twos. So then, eight people may sit in this way—

```
  × ×
×     ×
×     ×
  × ×
```

Two at each end, and Two at each side.

But if only one person sat at each end, how many would there be at each side? Then there would be three at each side, because $3 \times 2 = ?$ 6, and $6 + 2 = ?$ 8.

```
   ×
×     ×
×     ×
×     ×
   ×
```

That is how they would sit.

What does Units mean? I do not think you can have forgotten, but if you have, you must look in your book.

How many Units are there in Eight? There are eight Units in Eight. If there were not, it would not be Eight at all, but some other number.

Now say all the even numbers you have learned. 2, 4, 6, 8.

Now again. Once more.

Now say them backwards. 8, 6, 4, 2. Say them as quickly as you can.

2, 4, 6, 8. How many is each number more than the one before it? Each number is Two more than the one before it.

8, 6, 4, 2. How many is each number less than the one before it? Two less.

LESSON XXXI.

$2 \times 4 = ?$ 8. Then $4 \times 2 = ?$ 8. Yes, the eight people having dinner together might sit four on each side of the table if they liked, but then there would be no one to sit at either end. Set four bricks in one row, and four in another, and then you will see.

$8 \div 4 = ?$ 2. $8 \div 2 = ?$ 4. $8 - 2 = 6$. $8 - 6 = ?$ 2.

If you take Two from Eight, you have how many left? Six.

But if you take away Six, you keep the Two.

You can tell that that is right in another way. How many Twos are there in Eight? Four.

And in Six, how many Twos? Three.

So, if you subtract Six, which is three Twos, from Eight, which is four Twos, you must have one Two left, because $4 - 3 = 1$.

When you have only Two left, suppose you were to put the Six back again, how many would

there be? (pp. 61, 73.) Then there would be Eight $2+6=$? 8.

$8-3=$? 5. That is easy. Your fingers and the Roman VIII help you to remember that $8-3=5$.

$5+3=$? 8. $3+5=$? 8. As you have to add Five to Three to make Eight, if you subtract the Five again, there will be only the Three left by itself.

In one Eight, how many Twos? Four.

In two Eights, how many Twos? Four for one Eight, and four for the other Eight. $4+4=$? 8.

In two Eights, then, there are eight Twos.

Here we see again, that in Multiplication it does not matter which figure is put first (pp. 71, 84), $8 \times 2 = 2 \times 8$.

How many Fours are there in Eight? Two. In two Eights then there must be how many Fours? Four Fours.

In one Eight, four Twos and two Fours; in two Eights, eight Twos and four Fours.

Now you must write your Multiplication Table again.

$1\times2=2$	$1\times3=3$	$1\times4=4$
$2\times2=4$	$2\times3=6$	$2\times4=8$
$3\times2=6$		
$4\times2=8$		

You have put a new number to the first line,

the Two line. Then the Three line says, "Is there nothing for me?" And we say, "No, Three line, there is nothing for you; Eight is only Two more than Six, so we have not another Three to give you. You must have patience; perhaps your turn will come next; and just now we have begun a new line, the Four line, because we know that four times Two is Eight."

SUMS.

2	3	2	4
3	2	1	0
2	0	3	2
1	2	0	2
—	—	—	—

$3 + 5.$ $2 + 4.$ $2 + 5.$ $3 + 4.$

$8 \div 2.$ $8 \div 4.$

$6\overline{)\,8\,}$ $5\overline{)\,8\,}$ $3\overline{)\,8\,}$

LESSON XXXII.

HOLD up one hand. Now the other, but keep the thumb out of the way, and hold up only the fingers. Five on one hand and four on the other, how many does that make?—count. Nine.

Is Nine an even or an odd number? Nine is an odd number. How do we know that?

Because Nine is One more than Eight, which we know is an even number, and if we add One to an even number, the One makes an odd number (p. 64).

How must we write Nine? Show me how you think the Roman would write Nine. VIV? That is, you think he would write Five and Four more, because Nine is Four more than Five.

It is a good guess, but it is not quite right, and neither is VIIII. So many Ones look awkward. Do you remember how you had to wait to learn how to write the Roman IV? Well, now you must wait a little before you learn how to write the Roman Nine.

This is the Arabic Nine—9. Write it five times in your book, and leave a place for the Roman Nine.

Nine is One more than? Eight. Yes, 8 + 1 =? 9. So 1 + 8 =? 9 (pp. 27, 71).

Suppose there were eight children being drilled, and a sergeant to call out, "Quick March," "Right about face," and all the rest of it; how many persons would there be? Eight children + one sergeant = ? Nine persons.

The sergeant might make the children stand in two rows. How many children would there be in each row? How many Twos are there in Eight? Four Twos.

So there would be? Four children in each row.

Set eight bricks on the table in two rows,

and one tall one for the sergeant; he is the odd one you see.

Now the sergeant says he wants the children all to stand in pairs. How many pairs are there? Four pairs, for $8 \div 2 = ?$ 4. So now we have one One and four Twos of bricks.

Quick March.

Now put the odd brick, the sergeant you know, against one of the Twos. What have we now? One Three and three Twos. But $2 \times 3 = ?$ 6. And $6 \div 3 = ?$ 2.

You know there are always two Threes in Six. Now put those three Twos into two Threes. What have we now? Now we have three Threes.

So we see that there are three Threes in Nine.

Do you remember how Three grumbled because we could not give him another number for his line? Eight is only Two more than Six; but now that we have another One, that makes Three more than Six, and that is three Threes altogether.

So we see that though we cannot divide Nine by Two, we can divide it by? By Three.

Now say all the odd numbers you have learned. 1, 3, 5, 7, 9.

Can we divide the other odd numbers by any number? Let us try. We will begin with Seven. Two is no use you know. How do we know that? Because odd numbers cannot be divided exactly by Two; there is always the odd One over (p. 58).

Then let us try Three. How many Threes are there in Seven? Two, and one Unit over. Three will not do.

How many Fours are there in Seven? Only one Four, and three Units over, and only one Five and one Six.

So we cannot divide Seven exactly by any of these numbers; there are always some left over.

Now for Five. How many Twos are there in Five? Two, and one Unit over. How many

Threes? Only One, and two Units over, and only one Four. So Five is like Seven; it cannot be divided by any other number except One.

There is only one Two in Three, and one Unit over; and as for poor little One, we know quite well *that* cannot be divided.

No; 1, 3, 5, and 7, cannot be divided by any number except One. Of course all numbers are made up of Ones, and so they can be divided by One. $7 \div 1 = ?$ 7. $3 \div 1 = ?$ 3.

Now do you see that though Nine is an odd number it is different from 1, 3, 5, 7?

How is Nine different? Nine is different from 1, 3, 5, 7, because it can be divided by some other number, and 1, 3, 5, 7 cannot.

Those numbers which cannot be divided by some other number have a name of their own, they are called—Primes.

Are the Primes even or odd numbers? They must be odd, because? Because we can always divide even numbers by Two.

But there is just one even number which cannot be divided by any other number, except One of course. Which even number can that be? Why it must be Two; you cannot divide that by any other number. Two is the only even number which is a Prime. All other Primes are? Are odd numbers. So they are; all Primes except Two are odd numbers, but, as we found out, all odd numbers are not Primes.

So now when you meet a new number you must say—Pray, Master Number, are you odd or even? And if it be an odd number you must ask whether it be a Prime. And if it be not a Prime, you must ask by what numbers it can be divided. If you ask these questions, and take care to have the right answers, you will soon get to know enough about Giant Arithmos to make him begin to be your servant.

I hope you did not forget to say the three lines of Multiplication. Three-line must have patience until to-morrow, to have his new number written. There are so many other things to write.

First the Arabic 9—five times.

Then Primes—five times.

Then all the Arabic numbers up to Nine, the odd numbers as usual in red ink. Of course you *might* have ink of another colour for the Primes, but that would be rather awkward. The best way will be to mark the Primes by drawing a line round them, so 1 Shall you draw a line round Nine?

LESSON XXXIII.

How many Threes are there in one Nine? Three.

Then how many Threes are there in two Nines? Two Threes =? Six. So in two Nines there are? Six Threes.

How many of those Threes would it take to make one Six? Two Threes make one Six. Then how many Sixes are there in Six Threes? Two Threes make one Six, so there must be three Sixes in Six Threes.

Just half as many Sixes as Threes, because Three is half as much as Six; or we may say Six is twice as much as Three. So in two Nines there are Six? Six Threes, or three Sixes.

How many Twos are there in Nine? Four Twos, and one Unit over.

Then how many Twos are there in two Nines? Twice four Twos, that is? Eight Twos, and two Units over.

Why, that makes another Two; so there are nine Twos in two Nines. That is right; we must not forget that there is one Unit over in each of the Nines, so that as there are two Nines there are two Units over, and that, as you say, makes another Two.

How many Fours are there in Nine? Two, and one Unit over.

How many Fours in two Nines? Twice two Fours, that is four Fours, and two units over—one Unit over for each Nine.

If you take Four from Nine, how many will be left? You can tell that with your counting-machine. If you take away the four fingers of the second hand you will have the one hand still held up. That is Five. $9 - 4 = ?$ 5.

9 − 5 ? You see at once that that must be Four; your counting-machine helps you again, 9 − 5 = ? 4.

How many Fours are there in Nine ? Two. Then if you subtract one of the Fours, what will you have left ? The other Four and the odd One ; that makes Five.

And if, instead of subtracting Four, you subtract Five, that is one Four and the odd One, you will leave ? The other Four.

So you see that by looking at your counting-machine you can tell that 9 − 4 = ? 5; and 9 − 5 = ? 4 ; just by thinking about it. And thinking about the numbers is the best way of finding out Giant Arithmos' riddles.

9 − 6. You know what 9 − 5 is ? 4. And if you subtract One more than Five, that is Six, what will be left ? Three. 9 − 6 = ? 3.

You might find that out in another way. You know how many Threes there are in Nine ? Three. And in Six ? Two. Then if you subtract those two Threes you have left ? One Three. Yes, 9 − 6 = 3. 3 + 6 = ? 9.

Addition, you see, shows that Subtraction is right, because if you subtract a number and then add it again, you will have the same number as you had before you subtracted anything (p. 74).

9 − 3 = ? 6. Of course you know that, because you know that 9 = three Threes, and 6 = two Threes.

Say the odd numbers. 1, 3, 5, 7, 9. Now the even numbers. 2, 4, 6, 8. When you said those numbers you counted, not by Ones, but by? By Twos.

$1 + 2 = ?$ 3. $3 + 2 = ?$ 5. $5 + 2 = ?$ 7. $7 + 2 = ?$ 9. $2 + 2 = ?$ 4. $4 + 2 = ?$ 6. $6 + 2 = ?$ 8.

Now let us count by Threes. $3 + 3 = ?$ 6. $6 + 3 = ?$ 9. Yes, 3, 6, 9.

Now backwards. $9 - 3 = ?$ 6. $6 - 3 = ?$ 3. 9, 6, 3.

Now for our Multiplication—

$1 \times 2 = 2$	$1 \times 3 = 3$	$1 \times 4 = 4$
$2 \times 2 = 4$	$2 \times 3 = 6$	$2 \times 4 = 8$
$3 \times 2 = 6$	$3 \times 3 = 9$	
$4 \times 2 = 8$		

Write it in your book before you do these sums—

1	3	2	0	4
2	1	3	5	0
1	4	2	2	2
5	1	1	1	3

$1+3+4$. $2+2+5$. $2+1+3$. $5+2+1$.

$9 - 5$. $6 - 3$. $5 - 3$. $5 - 2$. $9 - 6$. $9 - 4$. $9 - 5$. $9 - 3$. $3 - 3$.

4) 9   6) 9   2) 9   7) 9

LESSON XXXIV.

×. What is that? The sign of Multiplication. Why is it put at the beginning of the lesson? To remind you to say your Multiplication Table. I am so afraid you may forget, and I do not want to have to write so many words every day, so I shall make a sign.

Hold up one hand. Now the other, only bend the thumb down. How many does your counting machine show now? Nine.

Now we will let that other thumb stand up straight; how many now? Nine and One more = ? Ten.

All the fingers on one hand = ? Five. All the fingers on both hands = ? Ten.

Then two Fives = ? Ten. $5 \times 2 = ?$ Ten. $2 \times 5 = ?$ Ten (pp. 71, 84).

What did we guess that the Roman did when he wanted to write Five? He made a figure which was to stand for? As many as the fingers on one hand. What do you think he would do when he wanted to write Ten? Make a new figure to stand for as many as all the fingers on both hands.

What was it to be? Perhaps he twisted his fingers about for some time and could not find a new figure, and so thought he must have two figures of V, one after the other, so—VV; but

that did not do very well, so he may have thought he would set one over the other, so—$\genfrac{}{}{0pt}{}{\mathsf{V}}{\mathsf{V}}$. That was rather awkward too; but at last—I do not know how he managed it—but he thought of a good way. He turned the bottom V upside down, so, $\genfrac{}{}{0pt}{}{\mathsf{V}}{\Lambda}$. There are the two Fives, you see, and they can be easily made with two strokes, one like this /, and the other across it, so—X, and then we can put little ornaments like those on, I—X.

Now you can tell how to write the Roman Nine. V, with I before it, stands for Four, because Four is One less than Five. So X, with I before it, in this way, IX, is the Roman Nine, because Nine is ? One less than Ten.

Let us turn back to Lesson 2 (p. 11) and read again what we said about the difference between Roman and Arabic figures. You know you have been able to find out some of the Roman figures for yourself, but you could not have found out the Arabic figures, for any one of them would do just as well for any other number. 9 might have stood for Six, if people had agreed to call it so; and so on with the others.

How many different kinds of figures have we used in writing the Roman numbers up to Eight? Only two—I and V.

Then how did we manage to make them mean so many different numbers? Partly by changing their places. IV, you know, means Four, and the

same figures are made to mean Six. How? By changing their places and putting the I after the V.

How many different kinds of Arabic figures have we used for the numbers up to Nine? Why, we have used nine Arabic figures, a new one for each number. But if we went on so, and had a fresh figure for every number, there would soon be so many figures that no one would be able to remember their names.

What must be done then about these Arabic figures? How did the Romans manage with so few figures? They changed their places.

Then that is how the Arabic figures must be managed. How? Change their places. Yes; let us see how it is done.

1. What figure is that? One. It only means one One or Unit when it stands alone; but if another figure is put after it, it no longer means one One or Unit, but one Ten.

Now when we want to write Ten, what figure must we put after the 1 to make it mean Ten?

Do we want to make it mean more than Ten? No; it is to mean just one Ten, and not any more. What figure do we use for Not any, or Not one? 0. Then let us put 0 after 1, so—10.

What does the 1 mean now? One Ten. And the Nought means that there are no Units to add to the Ten. How many Units are there in Ten?

When a figure stands alone it means Units; when another figure is put after it, it means? Tens.

So 1 was made to mean ten times as much by having a 0 put after it.

Units and Tens.

"Ah ha!" One says, "all you figures laughed at me because I was so little; but now I have grown ten times as large as I was before, and I am greater than any of you, greater even than Nine, who is the biggest of you all." One must not be *too* proud.

Now you have to write the Roman Nine in the place you left for it; to write the Roman X five times; and the Arabic 10 five times.

Is Ten odd or even? We know that it is

an even number because it is One more than Nine, which is an odd number. Besides, we know that in Ten there are just two Fives and nothing over; or five Twos and nothing over.

LESSON XXXV.

×. How many Fives are there in Ten? Two Fives in one Ten. Then you can tell what I want to know next; what is it do you think? How many Fives there are in two Tens. Well, can you tell?

There must be four Fives in two Tens. That is right—two Fives in one Ten, and four Fives in two Tens.

Then how many Fives are there in three Tens? Two Fives in each Ten, that will be six Fives.

Yes, and in four Fives? Four times Two is Eight. Eight Fives.

Dear me! what a great number; and you can go a little further, and tell how many Fives there are in five Tens. Why, there must be ten Fives in five Tens.

In one Ten how many Fives? Two. In two Tens? Four Fives. In three Tens? Six Fives. In four Tens? Eight Fives. In five Tens? Ten Fives.

$5 \times 2 = ?$ 10. $2 \times 5 = ?$ 10. Five Twos in one Ten.

How many more than Eight is Ten? Ten is Two more than Eight, and we know how many Twos there are in Eight; how many? Four. Yes, and one more Two of course makes five Twos.

How many Twos are there then in two Tens? Two Fives of Twos, that is ten Twos in two Tens. We could find that out quite easily by writing it, so — $2 \times 10 = 10 \times 2$ (pp. 71, 73); but it is much better to find it out by thinking. That is the way to get a little power over Giant Arithmos.

Say the even numbers as far as Ten. 2, 4, 6, 8, 10. Now backwards. 10, 8, 6, 4, 2. Say them very loudly. Now whisper, but not too softly.

How many Fours are there in Ten? Two Fours, and two Units over.

Then how many Fours are there in two Tens? Four Fours you say. But how many over? Two Twos over, one Two for each Ten.

But two Twos = ? Four. Why that is another Four; how many does it make? We said there were four Fours in two Tens, and something over, and now we find that what is over just makes another Four; so in two Tens there must be? Five Fours. At the beginning of this lesson we found out that there are four Fives in two Tens, and of course four Fives must equal five Fours. $4 \times 5 = 5 \times 4$ (p. 71).

How many Threes are there in Ten? Three Threes, and one Unit over.

Then how many Threes are there in two Tens? Six Threes and two units over. Yes, we cannot make another Three out of what is over of the Threes, as we did with the Fours.

$10-3=?$ One less than $10-2$, and we know what that is? 8. Then $10-3=?$ 7.

Put ten bricks on the table. Take three away. How many are left? Seven.

Put the three back again. How many are there now? Now there are ten again, as there were before you took the three away (pp. 73, 74).

Now take seven bricks away. How many are left? Three.

And if you put the seven back again of course there will be? Ten once more.

$10-3$? 7. $7+3=?$ 10. $10-7=?$ 3. $3+7=?$ 10.

$10-4$? How many Twos are there in Four? Two. And in Ten? Five.

Then if we subtract two Twos from five Twos, how many shall we have left? Three Twos; and that is? Six.

Besides, we know how many will be left if we subtract Five from Ten. How many? Five. And if we only subtract Four, which is One less than Five, of course One more will be left; we shall have Six left instead of Five.

But if we subtract Six from Ten, Six is One more than Five, so there will only be Four left.

$10-4=?$ 6. $10-6=?$ 4. $4+6=?$ 10. $6+4=?$ 10.

We must not forget to begin the new Multiplication line,

$1\times2=2$	$1\times3=3$	$1\times4=4$	$1\times5=5$
$2\times2=4$	$2\times3=6$	$2\times4=8$	$2\times5=10$
$3\times2=6$	$3\times3=9$		
$4\times2=8$			
$5\times2=10$			

Write it very nicely in your book.

SUMS.

				2
1	3	5		4
3	2	0	2	2
4	4	2	2	0
1	1	3	5	2

3) 10   5) 10   2) 10   6) 10   4) 10

$10-2$. $10-8$. $10-6$. $10-4$. $10-3$. $10-7$. $10-1$.

LESSON XXXVI.

×. Hold up your hands. We have counted all the fingers on these two hands; have you any more? Why no, of course not, no one has more than two hands. Then what are we to do? Can we get any more help from your counting-machine?

Yes, I think so. We will try to remember that we have counted one Ten, and begin to count another Ten. When we have finished, how many Tens shall we have counted? Two Tens.

Well now hold up one finger. Remember we have counted Ten before, so that is One more than Ten. We call One more than Ten? Eleven.

Now how would the Romans write Ten and One more? XI.

That is right, and it tells us what it means—Ten, and One more.

Now we must think how to write the Arabic Eleven. *How* many Tens have we to write? One Ten, and One more what? Another Ten? Oh no, a One or a? A Unit. So we write 1 for the Ten and another 1 for the Unit—11. One Ten and one Unit.

What kind of number is Eleven? An odd number? How do we know that? Because it is One more than an even number.

Do you remember what new question we have to ask the odd numbers, or must we turn back to Lesson 32? (p. 107) Ah, now you know. Master Eleven are you a? Prime. Eleven is rather rude; he says we may find out. I think we can manage it.

We know that it is of no use trying to divide an odd number by Two. Will Three do? $3 \times 3 = ?$ 9. How many more than Nine is Eleven? Only Two more, so Three will not do.

Well then, Four? $4 \times 2 = ?$ 8. Eleven is how many more? Eleven is Three more than Eight. So Four will not do; and as for Five, we know that there are two Fives in Ten, and Eleven is only One more than Ten; so it seems that Eleven must be? A Prime.

Do not forget to put a line round it when you write it in your book. Of course we do not always make lines round Primes, but I want you to do so that you may remember which they are.

$9 \div 2 = ?$ 4, and 1 over. So you may place four Twos of bricks, and let the odd one stand by itself. The sergeant has eight children to drill to-day.

Now let us set the nine bricks in threes, so—

× × ×
× × ×
× × ×

You see that counts three upwards, and downwards, and across; three every way, because there are? Three Threes in Nine.

Or we may set the bricks like this—

× × × ×
× × ×
× ×

There is a funny figure $4 + 3 + 2 = ?$ 9.

Can you tell which is the middle number in Nine? There are two Fours you know, and an odd One. What is the odd number which comes after Four? Five. So Five must be the

middle number in Nine; it has Four before it and Four after it.

What is the middle number in Eleven. In Eleven there are two? Two Fives; so Six must be the middle number, with Five before it, and Five after it.

Put eleven bricks on the table, two rows of five, with the odd one in the middle, that will be the sergeant with ten children, and you will see. Now let us try what sort of shapes we can make with Eleven. Here is one—

× × × ×
×   ×   ×
× × × ×

Here is another—

× × ×
× × ×
× × ×
× ×

Three Threes to make Nine, and Two more to make Eleven.

Here is another—

× × × ×
× × × ×
× × ×

Two Fours and Three again.

And here is one more—

× × × × ×
× × × ×
× ×

How have we made that? Five, and Four, and Two.

Write XI and 11 in your book, and then you can do some sums. I think you know very well of what numbers Eleven can be made—

			4	0	
	3	5	2	2	1
2	3	1	0	6	7
5	2	2	3	1	2
4	3	3	2	2	1

11 - 2. 11 - 6. 11 - 3. 11 - 9. 11 - 7. 11 - 8. 11 - 4. 11 - 5.

2) 11   4) 11   3) 11   5) 11

6) 11   8) 11   7) 11   9) 11

LESSON XXXVII.

×. What kind of number are we to learn about to-day. An even number, because it will be One more than an odd number.

How many fingers must you hold up? One more than for Eleven, that is two; and there is something we must not forget—what is that? That we have counted one Ten, so that now we are counting 10 + 2. We call that Twelve, as I dare say you know.

Now write Twelve in the Roman way. XII.

Now in the Arabic way. I am sure you can, at least I am *almost* sure you can show me how to write it. 12.

Once I said that the Roman figures told you what they meant, but that some of the Arabic figures did not tell you anything. But it is different now. We know that the figure on the left is worth ten times as much as it would be if it changed places with the figure on its right, so that 12 means? One Ten and two Units.

The Arabic figures tell what they mean now almost as plainly as the Roman figures, and that is very plainly indeed to people who have begun to find out some of the good Giant's riddles. People who know nothing about Giant Arithmos cannot understand what the figures say of course.

As Twelve is an even number, we know that it can be divided by what? By Two; we can always make two rows of bricks without an odd one in the middle when we have an even number.

How many would there be in each row if we had twelve bricks? When we had Ten there were? Five in each row. So when we have Twelve there must be? Six in each row. Yes. $6 \times 2 = ?$ 12. $2 \times 6 = ?$ 12.

Do you remember when we talked about one Six, we found out that there must be six Twos in two Sixes? (p. 90).

And we found out something else too—what was it? If there are two Sixes, how many Threes must there be. Two for each Six; that makes four Threes. $3 \times 4 = ?$ 12. $4 \times 3 = ?$ 12. Dear me, how many different ways there are of making Twelve. We can take two? Sixes. Or six? Twos. Or four? Threes. Or three? Fours.

Put two Sixes of bricks on the table, and then make them into Twos. Then into Threes. Then into Fours.

We have said before (pp. 71, 101) that in Multiplication it does not matter which figure we take first, and now you see that $2 \times 6 = 6 \times 2$, and $4 \times 3 = 3 \times 4$.

When you set the bricks in six Twos there were twelve bricks, and when you set them in two Sixes there were just twelve. For when you set the bricks in six Twos there really were two Sixes—

× ×
× ×
× ×
× ×
× ×
× ×

If you count the rows across, the bricks stand in Twos. If you count them lengthways, there are two rows of Six.

In the same way if you set the bricks in Fours there will be three rows, and you can

count them in Threes if you please, and there will be twelve all the time.

× × × ×
× × × ×
× × × ×

Our Multiplication Table will get on very fast to-day, and we shall have to begin a new line. What line will that be? The Six line.

$1 \times 2 = 2$	$1 \times 3 = 3$	$1 \times 4 = 4$
$2 \times 2 = 4$	$2 \times 3 = 6$	$2 \times 4 = 8$
$3 \times 2 = 6$	$3 \times 3 = 9$	$3 \times 4 = 12$
$4 \times 2 = 8$	$4 \times 3 = 12$	
$5 \times 2 = 10$		
$6 \times 2 = 12$		

$1 \times 5 = 5$	$1 \times 6 = 6$
$2 \times 5 = 10$	$2 \times 6 = 12$

How many Sixes must there be in two Twelves? Two Sixes in one Twelve, four Sixes in two Twelves.

How many Sixes in three Twelves? Two for each Twelve, that is, six Sixes in three Twelves.

How many Sixes in four Twelves? Eight. And in five Twelves? Ten Sixes in five Twelves.

And how many in six Twelves? In six Twelves of course there are twelve Sixes. You see again it does not matter whether you write 6×12, or 12×6 (p. 73).

How many Twos are there in one Twelve? Six. Then how many Twos are there in two Twelves? Twelve Twos. $2 \times 12 = 12 \times 2$.

How many Threes are there in one Twelve? Four. Then how many Threes in two Twelves? Eight, four Threes for each Twelve.

And how many Threes in three Twelves. Four Threes for each Twelve, so there must be twelve Threes in three Twelves. $12 \times 3 = 3 \times 12$.

How many Fours in one Twelve? Three.

How many Fours in two Twelves? Three Fours for each Twelve, six Fours in two Twelves.

And how many Fours in three Twelves? Three Fours for each Twelve makes nine Fours in three Twelves. Yes, and you can tell how many Fours there are in four Twelves. That is very easy; in four Twelves there are twelve Fours. $12 \times 4 = 4 \times 12$.

Do you know the names of the months in the year, and how many there are? Count—January, February, March, April, May, June, July, August, September, October, November, December. Twelve months in one year.

Now you must write the Multiplication Table. That will take some time. Then you may do these sums—

4	2			3	
2	5	2	1	0	4
3	0	7	4	6	2
0	3	2	3	1	3
1	2	1	1	2	2
—	—	—	—	—	—

$12-4$. $12-7$. $12-9$. $12-2$. $12-8$. $12-10$. $12-5$. $12-3$. $12-6$. $12-11$. $12-12$.

$12 \div 4$ $\quad 12 \div 6$ $\quad 12 \div 2$ $\quad 12 \div 3$

5) 12 $\quad$ 7) 12 $\quad$ 9) 12

LESSON XXXVIII.

×. Hold up three fingers. What do we mean by that? We mean Three more than Ten, because this is the second Ten we are counting now. What do we call Three more than Ten? Thirteen.

Teen means Ten. What does Thir mean, do you think? Thir means Three. Say Three over and over very fast, and presently you will find yourself saying something like Thir. So Thirteen means Three + Ten.

First the Roman figures told us what they meant. Then the Arabic figures began to tell us what they meant, and now the names of the numbers are beginning to tell us what they mean.

It is certainly very obliging of the names of the numbers, and will help us very much in adding and subtracting.

Thirteen! It hardly could mean anything except Three + Ten. It could not mean Four + Ten, neither could it mean Three + Nine.

But if you did not quite understand what Ten is, and what Three is, you could not well understand the meaning of the word Thirteen. When you learn to understand one of the Giant's little riddles perfectly, that helps you to understand others; and when you understand those others, you can go on and find out yet harder ones.

Roman XIII. Arabic 13. One Ten + three Units.

Is Thirteen odd or even? Odd. It is One more than? Than Twelve, which is? An even number. If we add One to an even number, we know that it makes an odd number (p. 82). But if we add more than One, any odd number we like, to an even number, what sort of number will that make? Ten is an even number, but when we add Three we make Thirteen, which is an? An odd number. Any odd number added to an even number makes an odd number, for the odd number always has an odd One.

But if we add Three to Nine, which is an odd number, what kind of number will *that* make? $3+9=$? 12. Is Twelve odd? No, Twelve is

an even number. In two odd numbers there are two odd Ones. If we add these two odd Ones, we make Two, which is an even number. So two odd numbers added together always make an even number. You can make out for yourself that when two even numbers are added together they make an even number.

Is Thirteen a Prime? By what numbers do we know that Twelve can be divided without leaving any over? By Two, or by Three, or by Four, or by Six.

Then if we divide Thirteen by any of those numbers, how many shall we have over? Just One, the odd One, you know.

$13 \div 5 = ?$ We know that two Fives make? Ten. And Thirteen is three Units more than Ten. So Thirteen, divided by Five, equals Two, and Three over.

But can we divide Thirteen by Seven? We know that there are two Sixes in Twelve. Thirteen is one Unit more than Twelve. If we add that Unit to one of the Sixes it makes Seven. So $7 + 6 = 13$, and $13 \div 7 =$ One, and Six over.

So it seems that Thirteen is a? A Prime.

$13 - 3 = ?$ 10. $13 - 10 = ?$ 3. $3 + 10 = ?$ 13. $10 + 3 = ?$ 13.

You see, as the name Thirteen tells us so plainly what it means, it is very easy to subtract Three or Ten from Thirteen, and so you can easily subtract Four or Eleven. $13 - 3 = ?$ 10.

$13-4=?$ 9. Of course, if you subtract One more, One less is left.

$13-10=?$ 3. $13-9=?$ 4. Then you subtracted One less than Ten, so One more was left. $13-11=?$ 2. $2+11=?$ 13. $4+9=?$ 13.

What is the middle number in Thirteen? If you put thirteen bricks on the table in two rows, with the odd one in the middle, how many will there be in each row? Six. Then the odd one will be number? Number Seven.

So the middle number of Thirteen is? Seven, with Six before, and Six after it.

$13-6=?$ 7. $13-7=?$ 6. $13-8=?$ 5. $13-5=?$ 4. $6+7=?$ 13. $7+6=?$ 13. $8+5=?$ 13. $5+8=?$ 13.

Now let us see what patterns or shapes we can make with thirteen bricks or anything else.

× × × ×	× × ×	× × × × ×	× × × × × ×
× × × ×	× × ×	× × × ×	× × ×
× × × ×	× × ×	× × ×	× × × ×
×	× × ×	×	
	×		

			0	5
	2	1	3	0
4	5	0	6	2
7	2	3	2	4
1	4	9	1	2
—	—	—	—	—

6) 13　　4) 13　　8) 13　　2) 13

7) 13   9) 13   3) 13

13 − 7. 13 − 4. 13 − 6. 13 − 2. 13 − 5. 13 − 8. 13 − 9. 13 − 5.

Of course you will write the Roman and Arabic Thirteens in the usual way.

LESSON XXXIX.

×. To-day we must have four fingers held up, because we are going to talk about Four more than the one Ten we have counted. What do we call Four more than Ten? Fourteen.

Does not that name tell us quite plainly what it means? It *could* not mean Two more than Ten, or Three more than Ten, or Five more than Ten, could it? As you know that teen means Ten, you can tell at once that Fourteen means? 4 + 10.

And you can tell, I am sure, how it is written in Roman figures? XIV.

And in Arabic? 14. That is right, one Ten. And? Four units. 4 + 10 = ? 14. 10 + 4 = ? 14. 14 − 4 = ? 10. 14 − 10 = ? 4.

Is Fourteen an even number? Yes, you are quite sure that it is. Fourteen is One more than? Than Thirteen. And Two more than? Than Twelve. Then how many Twos are there in Fourteen? In Twelve there are six Twos, so in Fourteen there are? Seven Twos. 2 × 7 = ? 14. 7 × 2 = ? 14.

In two Fourteens, how many Sevens? Four Sevens in two Fourteens.

How many Sevens in three Fourteens? Six Sevens in three Fourteens, two Sevens for each Fourteen.

Find out by thinking how many Sevens there are in four, five, six, and seven Fourteens.

Suppose we have eleven bricks, how many more shall we want to make Fourteen? If we had ten, we should want? Four more. But Eleven is One more than Ten, so we only want? Three more. $11+3=$? 14. $3+11=$? 14. $14-3=$? 11. $14-11=$? 3.

If we take Ten from Fourteen, Four are left, but if we only take Nine, how many will be left? Five, because Nine is One less than Ten. $14-9=$? 5. $14-5=$? 9. $5+9=$? 14. $9+5=$? 14. $14-7=$? 7. $14-6=$? 8. $14-8=$? 6. $6+8=$? 14. $8+6=$? 14.

Now I really think we may go on to another number, because you are getting to understand so much about the numbers, that we need not stay so long over each one as we were obliged to do at first. Is not that nice?

What is the next number to Fourteen? Fifteen. What does that mean? Why of course, it means Five + Ten. If you say Five-teen over and over again very quickly, Five will soon sound like Fif.

Hold up your hand. Why we have got half

through another Ten, we *are* getting on, and no doubt you can write XV, 15.

One Ten and five Units. $5+10=?$ 15. $10+5=?$ 15. $15-5=?$ 10. $15-10=?$ 5.

What kind of number is Fifteen? An odd number. Yes, but is it a Prime? $5+10=?$ 15. But how many Fives are there in Ten? Two Fives. Then how many Fives are there in Fifteen? Three Fives.

So Fifteen can be divided by Five. It is not a Prime, and will not want a line round it in your book.

We have learned some new multiplication— $5\times3=?$ 15. $3\times5=?$ 15. $15-3=?$ 12.

You remember there are four Threes in Twelve, and in Fifteen there are? Five. $3+12=?$ 15. $15-9=?$ 6. That is One more than $15-10$.

$15-11=?$ 4. That is One less than $15-10$.

You see in all these numbers, which tell us so plainly what they mean, subtraction is very easy. We know at once how many will be left if we subtract Ten. The Units will be left, and the name of the number tells us what they are. Then if we subtract One more than Ten, that is Eleven, One less will be left than if we subtracted Ten.

And if we subtract One less than Ten, that is Nine, One more will be left than if we subtracted Ten.

If we subtract the Units, the Ten will be left. $15-5=?$ 10. $15-6=?$ 9. $15-4=?$ 11.

What is the middle number in Fifteen? Take fifteen bricks and make two rows. Put the odd brick, the sergeant, at one end between them. The middle number is? Eight. $15-8=?$ 7. $15-7=?$ 8. $7+8=?$ 15. $8+7=?$ 15.

$1\times2=2$	$1\times3=3$	$1\times4=4$
$2\times2=4$	$2\times3=6$	$2\times4=8$
$3\times2=6$	$3\times3=9$	$3\times4=12$
$4\times2=8$	$4\times3=12$	
$5\times2=10$	$5\times3=15$	
$6\times2=12$		
$7\times2=14$		

$1\times5=5$	$1\times6=6$	$1\times7=7$
$2\times5=10$	$2\times6=12$	$2\times7=14$
$3\times5=15$		

What a great deal you have to write in your book. XIV, 14, XV, 15; then all that multiplication; then all the even numbers we have talked about; then all the odd numbers. We must wait for the next lesson till you have had time to get through all this writing.

LESSON XL.

×. Let us see if you can say the even numbers up to Fourteen quickly. 2, 4, 6, 8, 10, 12, 14.

Now the odd numbers. 1, 3, 5, 7, 9, 11, 13, 15.

Now both sets backwards quickly.

Let us try to count by Threes. $3+3=?$ 6. $6+3=?$ 9. $9+3=?$ 12. $12+3=?$ 15. 3, 6, 9, 12, 15.

Over and over again till you can say it quickly, and then backwards in the same way.

$4+4=?$ 8. $8+4=?$ 12. 4, 8, 12. Fours will not take long to say backwards and forwards.

$5+5=?$ 10. $10+5=?$ 15. 5, 10, 15. Fives, too, can be quickly said.

In our last lesson we found that we were half-way through another Ten, so to-day for the next number to Fifteen you will have to hold up one hand and one finger of the other hand. We call the next number to Fifteen, Sixteen.

What does Sixteen mean? Sixteen means Six + Ten. Six Units and one Ten.

Units means Ones. $6+10=?$ 16. $10+6=?$ 16. $16-10=?$ 6. $16-6=?$ 10.

Let me see you write Sixteen. XVI, 16.

You see the Roman figures say—Ten+Five+One. $10+5+1=?$ 16.

The Arabic figures say—one Ten and six Units.

What kind of number is Sixteen? An even number. Then of course we have another Two, because we have added One to Fifteen, in which there was an odd One, and the two Units make? Another Two. So they do. There is a rhyme for you.

How many Twos are there in Sixteen? There were seven Twos in Fourteen, so there must be? Eight Twos in Sixteen. $2 \times 8 = ?$ 16. $8 \times 2 = ?$ 16.

But if we have eight Twos, do you not think we must have something else? We know we have two Eights, and in each Eight there are? Four Twos. Yes, or two? Two Fours. Then if there are two Fours in one Eight, how many Fours are there in two Eights? Four Fours. So there are.

Put eight Twos of bricks on the table. Now make them into Fours.

And now into Eights. You have just sixteen bricks all the time, but sometimes you made them stand in? Sometimes in Twos or pairs, sometimes in Fours, and sometimes in Eights. $2 \times 8 = ?$ 16. $4 \times 4 = ?$ 16. $8 \times 2 = ?$ 16 (p. 71).

Now let us count again by Fours. $4 + 4 = ?$ 8. $8 + 4 = ?$ 12. $12 + 4 = ?$ 16. 4, 8, 12, 16. Now count backwards, 16, 12, 8, 4.

$16 - 4 = ?$ 12. $16 - 5 = ?$ 11. $16 - 8 = ?$ 8. $16 - 7 = ?$ 9. $16 - 10 = ?$ 6. $16 - 9 = ?$ 7. $16 - 6 = ?$ 10. $16 - 5 = ?$ 11. $16 - 13 = ?$ 3.

How many Tens are there in Sixteen? One Ten. And how many Tens in Thirteen? One Ten.

Do you see, in subtracting Thirteen from Sixteen we know at once that no Ten will be left, so all we have to do is to find out the difference in the Units. $6 - 3 = ?$ 3. So $16 - 13 = 3$.

$16 - 3 = ?$ 13. Sixteen has one Ten in it, but Three has no Ten; so this time Ten will be left, because we have only to subtract three Units. $6 - 3 = ?$ 3. So $16 - 3 = ?$ 13.

Place your sixteen bricks in one Ten and one Six. Now take three away. You see you did not touch the Ten, only the Units.

Now put the three bricks back to their place. $3 + 13 = ?$ 16.

Now take thirteen away. You have to take all the Ten and three of the Units, because Thirteen is one Ten + three Units.

I want to play a little game. Will you play with me if you please? I will be Tens and you shall be Units. When I say a number which has a Ten in it, I must hold up one finger for the Ten, and you must hold up as many fingers as there are Units.

Now then I say Sixteen, and hold up one finger. That means one Ten, because you know my name just now is Tens. So now Units, how many fingers must you hold up? Six, because beside the Tens there are six Units in Sixteen.

Now I say Fourteen, now Thirteen, now Fifteen, and now Twelve. How many fingers must Units hold up when I say Twelve? Two, because Twelve is Two more than Ten.

Now I say Nine, and Units holds up nine fingers, but what must Ten do? Ten must not

even hold up one finger, because there *are* no Tens in Nine.

Now I say Ten, so Ten may hold up one finger, but what must Units do? Ten means just one Ten, with no Units added to it, so this time Units must not hold up even one finger.

Now for the Multiplication Table, which grows and grows, and every time it grows a little bit you must write it in your book as nicely as you possibly can.

$1\times2=\ 2$	$1\times3=\ 3$	$1\times4=\ 4$	$1\times5=\ 5$
$2\times2=\ 4$	$2\times3=\ 6$	$2\times4=\ 8$	$2\times5=10$
$3\times2=\ 6$	$3\times3=\ 9$	$3\times4=12$	$3\times5=15$
$4\times2=\ 8$	$4\times3=12$	$4\times4=16$	
$5\times2=10$	$5\times3=15$		
$6\times2=12$			
$7\times2=14$			
$8\times2=16$			

$1\times6=\ 6$	$1\times7=\ 7$	$1\times8=\ 8$
$2\times6=12$	$2\times7=14$	$2\times8=16$

SUMS.

2					
0	1	4	6		4
3	2	5	0	3	3
5	9	2	7	5	7
4	3	1	3	8	2
—	—	—	—	—	—

16 - 4. 16 - 5. 16 - 12. 16 - 13.
16 - 9. 15 - 12. 16 - 11. 15 - 9.

4) 16 8) 16 2) 16 7) 16 9) 16

LESSON XLI.

×. To-day I want you to learn a new word. It is quite a long time since you learned a new word, but the one I am going to tell you is so very useful that I think it is time for you to know it, and its meaning. The word is—Factor.

Factor comes from *Facio*, which is a Latin word, and means—I make.

So it seems as if Factor had something to do with making, and so it has. $2 \times 2 = ?$ 4. Then it takes two Twos to *make* Four. So we say that Two is a Factor of Four.

It looks as if we might almost say that Factor means Maker, or a number that makes; but if we say that perhaps we shall make a mistake. $2 + 3 = ?$ 5. Then is Two a Factor of Five? No; because we only take Two once, and then we take another number to make Five.

A Factor is a number taken twice or several times to make another number; or we may say that the factors of any number, are those numbers by which it may be divided exactly and leave nothing over.

So all numbers are factors of some other

numbers, because all numbers may be taken twice or several times to make other numbers.

There are some numbers which have no factor except One. What do we call such numbers? Think, for you know quite well, for we have often tried to find factors for those numbers. These must be odd numbers, because, as you know, every even number bigger than Two has Two for a factor; that is, it can be made by taking the right number of Twos; but some odd numbers and one even number, that is Two, cannot be divided by any number except One, and are called? (p. 106) Primes. Yes; the primes have no factor except One.

What are the factors of Six? Two, because there are three Twos in Six. And what else? Three, because there are two Threes in Six.

What are the factors of Eight? Two again, because Eight is an even number; and not Three this time, but Four.

And what factor has Nine? Not Two, because Nine is an odd number, but Three, for there are three Threes in Nine (p. 105).

The factors of Twelve are? Two and Six. And what else? What other numbers can we use to make Twelve? (p. 124). Three and Four.

Is not Twelve rich in factors? Say them again. Two and Six, and Three and Four.

Now you can tell the factors of Fourteen? Two and Seven. Of Fifteen? Three and

Five. Of Sixteen? Two, and Eight, and Four (p. 136).

Sixteen is not so rich in factors as Twelve, but it is richer than most of the numbers we have had.

Say the even numbers up to Sixteen backwards and forwards quite quickly, and then say the odd numbers up to Fifteen in the same way.

Write Factor five times, and then write all the numbers up to Sixteen in Roman and Arabic figures, and mark the Primes.

LESSON XLII.

×. The last number we learned was? Sixteen. One Ten and six Units. What is the next number? Seventeen.

Now I will be Tens again and hold up one finger. How many must Units hold up? Seven, for Seventeen is one Ten and seven Units. Now you can write it. XVII. 17.

What do the Roman figures say? They say Ten + Five + Two, and that equals? Seventeen.

And the Arabic figures? They say one Ten + seven Units.

What kind of number is Seventeen? An odd number. Is it a Prime? Let us find out if it has any factors. We know that Two will not do; will Three? $3 \times 5 = ?$ 15. And Seventeen is how many more than Fifteen? Two

more. So Three will not do, and we know besides that another number will not do; what number is that? Why Five, of course; there are three Fives in Fifteen, and Seventeen is only Two more than Fifteen.

Shall we try Four? Oh, that is of no use, for $4 \times 4 = ?$ 16. And Eight will not do either, for $8 \times 2 = ?$ 16. As for Seven, $7 \times 2 = ?$ 14. So Seventeen must be a Prime, for it has no factor except One.

What is the middle number in Seventeen? You can find that out I think without the bricks. In Sixteen there are two? Two Eights. So Nine must be the middle number in Seventeen.

If you were counting Seventeen, when you came to Nine you would have counted Eight before, and have Eight more to count. Try it with the bricks if you like. $9+8=?$ 17. $8+9=?$ 17. $17-8=?$ 9. $17-9=?$ 8.

I have shown you before how easy it is to subtract from these *teen* numbers (p. 133). They tell us by their names how many Units they have besides the *teen* or Ten, so if we want to subtract Ten we know that we shall have just the Units left. $17-10=?$ 7.

If we subtract the Units, what is left? The Ten. $17-7=10$.

If we subtract One more than the Units, shall we have the whole Ten left? No, we shall have

to take one Unit out of the Ten, so only? Only Nine will be left. $17 - 8 = ?$ 9. $17 - 10 = ?$ 7.

If we subtract One more than Ten, we shall have to take away not only the Ten but one of the Units, so only? Only Six will be left. $17 - 11 = ?$ 6.

And if we subtract One less than Ten? Then we shall have the seven Units left, and one Unit out of the Ten; that is, we shall have? Eight left. $17 - 9 = ?$ 8.

If you want to find out how much any number of Units makes when added to Ten, you know that is very easy. If it is one Unit added to Ten, that makes? Eleven. If two Units are added to Ten, that makes? Twelve.

And for all the rest of the Units, three, four, five, six, seven, eight, or nine Units, you just add teen to their names if you want to add Ten.

So if you wish to add Nine to some other number of Units, you must say to yourself, If I added Ten to these Units it would be so much, and of course if I add Nine it will be One less, because Nine is One less than Ten.

Then if you wish to add Eleven to some number of Units, you know that it will make One more than if you only added Ten, because Eleven is One more than Ten. So Nine, Ten, and Eleven, are very easy numbers to add to any number of Units.

$6 + 10 = ?$ 16. $6 + 11 = ?$ 17. $6 + 9$

=? 15. 2 + 9 = ? 11. 3 + 11 = ? 14. 5 + 9 = ? 14. 3 + 9 = ? 12. 6 + 11 = ? 17. 4 + 11 = ? 15. 4 + 9 = ? 13.

Now this is one of our Giant's little riddles. If you did not understand the meaning of Units and Tens you could not find out this riddle. Presently it will help you to find out harder riddles, but if you tried to find them out without knowing all about these little things, what would Giant Arithmos do? He would say—"No, no, silly child, go back; I will not let you know my big riddles if you will not take the trouble to find out the little ones."

Write XVII, and 17, each five times.

SUMS.

3	2		7	1	4
4	5	1	2	0	2
6	3	9	0	2	0
2	4	4	1	3	2
1	3	2	6	11	9

17 - 9. 17 - 4. 17 - 10. 17 - 3. 17 - 11. 17 - 5. 17 - 12. 17 - 6. 17 - 13. 17 - 8.

3)17 6)17 4)17 5)17 12)17

9)17 13)17 10)17 2)17

8)17 14)17 15)17

LESSON XLIII.

×. What comes next to Seventeen? Eighteen. Then Tens must hold up one finger, and Units? Eight fingers. Eighteen = 8 + ? 10.

Now let us write Eighteen, and see what the figures tell us. XVIII says 10 + 5 + 3. 18 says one Ten + eight Units.

Is Eighteen an even number? Yes. Then we can tell one of its factors, which is that? Two, because all even numbers except Two itself have Two for a factor (p. 140).

How many Twos are there in Eighteen? In Sixteen there are? Eight. Then in Eighteen there are? Nine Twos. Or? Two Nines. 2 × 9 = ? 18. 9 × 2 = ? 18.

What factor has Nine? (p. 105). Three. 3 × 3 = ? 9.

Then how many Threes are there in two Nines? Three Threes in one Nine, six Threes in two Nines. So 3 × 6 = ? 18. 6 × 3 = ? 18.

Now let us see. 18 = ? 2 × 9, or 9 × 2, or 3 × 6, or 6 × 3.

Put eighteen bricks on the table, first in nine Twos, then in two Nines, then in six Threes, and then in three Sixes.

Now say again all the factors of Eighteen? Two, Three, Six, Nine. Eighteen has four factors. What number have we found before with four

factors, and what are they? (p. 140). Twelve has four factors—Two, Three, Four, and Six.

Now count by Twos all the even numbers to Eighteen backwards and forwards quite fast.

And now we will count by Threes. $3 + 3 = ?$ 6. $6 + 3 = ?$ 9. $9 + 3 = ?$ 12. $12 + 3 = ?$ 15. $15 + 3 = ?$ 18. 3, 6, 9, 12, 15, 18.

I hope you can manage that easily, and say it quickly backwards and forwards over and over again.

For fear you should forget, we will say the Fours too. $4 + 4 = ?$ 8. $8 + 4 = ?$ 12. $12 + 4 = ?$ 16. 4, 8, 12, 16. Backwards, 16, 12, 8, 4.

Now the Fives. $5 + 5 = ?$ 10. $10 + 5 = ?$ 15. 5, 10, 15. Backwards, 15, 10, 5.

And the Sixes, now we have three of them. $6 + 6 = ?$ 12. $12 + 6 = ?$ 18. 6, 12, 18. And backwards, 18, 12, 6.

$18 - 10 = ?$ 8. $18 - 9 = ?$ 9. $18 - 11 = ?$ 7. $18 - 6 = ?$ 12.

Twelve equals two Sixes, you know; and Eighteen equals three Sixes. $18 - 12 = ?$ 6. If you take two Sixes from three Sixes, only one Six is left. $18 - 15 = ?$ 3. $18 - 13 = ?$ 5. Because, of course no Ten is left, and $8 - 3 = ?$ 5 (p. 136).

This is a short lesson, but there is a good deal to write. First of course Roman XVIII, Arabic 18.

Then write: The Factors of 18 are—I need not tell you what they are; you know them and can write them.

Then write all the even numbers to Eighteen.

Then the Threes.

SUMS.

1	2			1		0
4	3	3	3	2	5	2
6	2	2	8	3	3	4
5	9	7	2	5	2	3
1	2	6	4	7	8	9

18 - 16. 18 - 6. 18 - 8. 18 - 11. 18 - 4. 18 - 14. 18 - 7. 18 - 12. 18 - 15. 18 - 2. 18 - 5. 18 - 9. 18 - 3. 18 - 13.

$18 \div 6$. $18 \div 2$. $18 \div 9$. $18 \div 3$.

7) 18 5) 18 8) 18 12) 18 4) 18

LESSON XLIV.

×. Another new number to-day, and Units must hold up? Nine fingers, while Tens holds up one.

And what do we call this number, do you know? Nineteen. Yes, the name tells what it means, just like the other *teens*. Nineteen means 9 + 10.

And now we can write it. Though we only want one more number to make another Ten, we need not wait to write Roman Nineteen, as we had to wait before we could write Four and Nine. You see we know how to write Nine already, and have only to write Ten before it. XIX.

What do the Roman figures say? Ten + one less than another Ten. And the Arabic? Ten + Nine.

"What kind of number are you, Nineteen?" "I am an odd number." "Of course we know that you are an odd number Nineteen. Why do you tell us what we know already? You are One more than Eighteen, which is an even number, so of course you are an odd number. But we want to know if you are a Prime?" "Ah, then," says Nineteen, "you must find that out; my master, Giant Arithmos, likes people to find out his riddles themselves."

Well, that is quite right, so now let us see if Nineteen has any factors. We know that it cannot have the same factors as Eighteen, because it is only One more than that number.

What are the factors of Eighteen? (p. 145.) Three, Six, Nine. Then, if we divide Nineteen by any of those numbers there will be One over, so those numbers are not factors of Nineteen.

Shall we try Four? $4 \times 4 = ?$ 16. How many more than Sixteen is Nineteen? Nine is Three more than Six, so Nineteen is? Three

more than Sixteen. So Four is not a factor of Nineteen.

How will Five do? $5 \times 3 = ?$ 15. Well, we know that we have not another Five yet, because if we had, we should have another Ten, for two Fives make Ten. Our counting-machine and the Roman figures tell us that we want One more to make another Ten. How many more than Fifteen is Nineteen, then? Nineteen is Four more than Fifteen. So Five is not a factor of Nineteen.

Then let us try Seven. $7 \times 2 = ?$ 14. Nineteen is—how many more than Fourteen? I hope you know that very well, but if you are not quite sure, just look at your hands and see how many are left when you take Four from Nine. Then you can tell at once that Nineteen is Five more than Fourteen. So Seven is not a factor of Nineteen, for Nineteen equals only Five more than twice Seven.

$8 \times 2 = 16$. So Eight cannot be a factor of Nineteen. So it seems that Nineteen is a Prime.

$19 \div 6 = ?$ Three and One over. But there is another word used in Arithmetic for over, or left over. When one number does not divide another number exactly, we call the number left over the Remainder.

When you divide Primes, there is always a Remainder. That is not a very hard word to remember, is it?

Put seven bricks on the table. Now give me one Two, take one Two yourself, and put one Two on a chair.

You have divided the Seven into three parts, but you could not do it exactly, and one brick remains on the table. The One which remains is called? The Remainder.

Put the six bricks on the table again; add them to the Remainder. Now we have? Seven again.

Subtract Four. What is the Remainder now? Now the Remainder is Three.

Subtract Three more. What is the Remainder now? Now the Remainder is Nought.

Write Roman XIX and Arabic 19—five times.

Write Remainder = left over—five times.

SUMS.

3	1			2	
2	5	0	5	9	3
3	4	8	2	4	8
4	7	7	9	1	3
6	2	4	3	2	5
—	—	—	—	—	—

19 − 7. 19 − 10. 19 − 9. 19 − 11. 19 − 5. 19 − 12. 19 − 3. 19 − 13. 19 − 15. 19 − 17. 19 − 8. 19 − 18. 19 − 2. 19 − 14. 19 − 4. 19 − 19.

Write the odd numbers as far as Nineteen, and mark the Primes.

LESSON XLV.

×. What number are we to have to-day? Two Tens. Then how many fingers must Units hold up. *Ten* must hold up two fingers, but Units not one.

But if we want to show how many Units there are in this new number, I must be Units as well as you, and we must both hold up all our ten fingers.

What do we call two Tens? We call it Twenty. Before we talk of what the name means, let us see what we know about Twenty.

We know two of its factors at any rate. What are they? Two and Ten. Yes, $10 \times 2 = ?$ Twenty. $2 \times 10 = ?$ Twenty.

Now Twen means Two. I daresay you have heard the word Twain, and know that it means Two. Twain and Twen sound much alike. Ty means Ten. What does Teen mean? Teen means Ten also.

Then would it do to say Twen-teen instead of Twen-ty? Oh no; that would not do at all. When Teen is put after a number, what does it mean? *Teen* means that Ten is to be *added* to the number after which it is put.

But *ty* means that the number after which it is put is to be *multiplied* by Ten, which is quite a different thing.

If we were to say Twenteen, it would mean? $2+10$. What do we call $2+10$? Twelve. Yes, we have a name for that number already. When we say Twenty, we mean? 2×10. And $2\times 10=$? Twenty.

If I were to promise to give you some fine rosy apples, or some nice toys, would you rather have a *teen* number, or a *ty* number? I think I know which you would choose. You would say, "A *ty* number, if you please, because a number multiplied by ten is much more than a number added to ten."

Tell me again what are the factors of Twenty? Two and Ten. But what are the factors of Ten? Two and Five. There are two Fives in every Ten.

Then how many Fives are there in Twenty? There must be Four. So there are.

Let us both hold up our hands, and then we shall see the four fives of fingers in the two Tens. $5\times 4=$? Twenty. And $4\times 5=$? Twenty (pp. 71, 73).

Let us hold up our hands again, and see that if we do not count the thumbs, we have each two Fours of fingers; and as you have two thumbs, and I have two thumbs, that makes the fifth Four. Twenty$\div 4=$? 5. Twenty$\div 5=$? 4.

We have found so much to say about Twenty that you have not yet written it, but I dare say you are quite ready to do so. How did the

Romans write two Ones ? II. Then how would they write two Tens ? XX.

You see the Roman figures as usual speak very plainly. When once we know that X stands for Ten, we shall hardly think that XX can stand for anything except Ten + Ten.

Now for the Arabic figure. When we mean one Ten, we write 10, which means ? One Ten and no Units.

So when we mean two Tens, we write 20, which means ? Two Tens and no Units.

How many Units are there in every Ten ? Ten Units in every Ten. Suppose we have one Ten + eight Units, it will take just as many more Units to make another Ten, as it would to make one Ten if we had only eight Units.

If we had only eight Units, how many should we want to make one Ten ? Two Units more.

So, if we have 10+8 Units, that is Eighteen, how many shall we want to make another Ten ? Two more Units to make two Tens.

Now count by Twos up to Twenty. You know how to do that so well, that I am sure I need not write it for you. I hope you can count so far by Twos almost as easily as by Ones.

To count by Fours is not quite so easy, but you can manage it for a little way. 4+4=? 8. 8+4=? 12. 12+4=? 16. 16+4=? 20. 4, 8, 12, 16, 20. Now backwards, 20, 16, 12, 8, 4.

Now, Fives. $5+5=?$ 10. $10+5=?$ 15. $15+5=?$ 20. 5, 10, 15, 20. And backwards, 20, 15, 10, 5.

Can you tell what is the middle number in Nineteen? $19-9=?$ 10. $10-1=?$ 9. So Ten must be the middle number, because when you come to Ten in counting Nineteen you have counted how many before? Nine before, and have Nine more to count after Ten.

What a great deal of Multiplication we have to write. Our Table grows very fast when we come to a number which has several factors. *What* are the factors of Twenty? Two, Four, Five, and Ten.

$1\times2=2$	$1\times3=3$	$1\times4=4$	$1\times5=5$
$2\times2=4$	$2\times3=6$	$2\times4=8$	$2\times5=10$
$3\times2=6$	$3\times3=9$	$3\times4=12$	$3\times5=15$
$4\times2=8$	$4\times3=12$	$4\times4=16$	$4\times5=20$
$5\times2=10$	$5\times3=15$	$5\times4=20$	
$6\times2=12$	$6\times3=18$		
$7\times2=14$			
$8\times2=16$			
$9\times2=18$			
$10\times2=20$			

$1\times6=6$	$1\times7=7$	$1\times8=8$	$1\times9=9$
$2\times6=12$	$2\times7=14$	$2\times8=16$	$2\times9=18$
$3\times6=18$			

SUMS.

	0			1	3
2	2	1		3	2
7	4	2	3	5	5
9	6	9	6	4	4
2	3	5	7	5	2
0	4	3	3	2	4

$20-5.$ $20-15.$ $20-8.$ $20-18.$ $20-4.$ $20-14.$ $20-7.$ $20-17.$ $20-3.$ $20-13.$ $20-2.$ $20-12.$ $20-6.$ $20-16.$ $20-9.$ $20-19.$ $20-20.$ $20-0.$

$20\div5.$ $20\div2.$ $20\div4.$ $20\div10.$

12) 20 9) 20 7) 20 6) 20

3) 20 8) 20

LESSON XLVI.

×. What comes next to Twenty; what do we call One more than Twenty? Twenty-one.

The name of this number tells us quite plainly what it means. Twenty means? 2×10. So Twenty-one says, "I am One more than Two Tens."

I think you can write Twenty-one in both

ways. Roman XXI. What does that say? Ten + Ten + One.

Arabic 21. And that says? Two Tens + One.

What kind of number is Twenty-one? An odd number. Yes, but is it a Prime? What factors do we know that Twenty-one cannot have? The same factors as Twenty. And those are? Two, Four, Five, and Ten. Then let us try Three.

Say the Three line of Multiplication. $3 \times 6 = ?$ 18. How many more than Eighteen is Twenty-one? Why, Twenty-one is Three more than Eighteen.

Then how many Threes are there in Twenty-one? Seven. $3 \times 7 = ?$ 21. And $7 \times 3 = ?$ 21. So 21 is not a Prime.

You know that there are two Tens in Twenty-one, so you can easily tell what is the middle number in Twenty-one. It must be the number next to Ten, and that is? Eleven. Yes; when you come to Eleven you have counted Ten, and must count Ten more to make Twenty-one.

Now let Tens and Units hold up their fingers for Twenty-one. Tens must hold up? Two fingers, because there are two Tens in Twenty-one.

And Units? Units must hold up one finger, because, beside the two Tens, there is one Unit in Twenty-one.

Now Units may hold up another finger.

What does that mean? Two Tens and two Units. What do we call two Tens and two Units? The name of the number is Twenty-two. We know at once how to write it. Roman—XXII. Those figures say? Ten+Ten+Two.

Arabic—22. And the Arabic figures say? Two Tens+two Units.

What kind of number is Twenty-two? An even number. Then we know one of its factors—what is that? Two (pp. 140, 145). And how many Twos are there in Twenty-two? In Twenty there are? Ten (p. 151). So in Twenty-two there must be? Eleven. $2 \times 11 = ?$ 22. $11 \times 2 = ?$ 22. $22 \div 2 = ?$ 11. $22 \div 11 = ?$ 2.

Has Twenty-two any other factors besides Two and Eleven? We know of two factors which it cannot have—which are they? Three and Seven. Because? Because those are the factors of Twenty-one.

Twenty-two is only Two more than Twenty, so that those two numbers could not have the same factors either. Except one little factor—what is that? Two, because all even numbers have Two for a factor (p. 140). So neither Three, nor Four, nor Five, nor Seven, nor Ten, can be a factor of Twenty-two.

Has Eleven any factors, or Two? No, neither Eleven nor Two has any factors.

So Twenty-two has only two factors, Eleven and Two.

$22-10=?$ 12. Yes, Twenty is two Tens + two Units. So if we subtract one of the Tens, we shall have only one Ten and two Units remainder.

If we subtract Ten we do not alter the number of Units. $12-10=?$ 2.

So we have subtracted first one Ten and then the other, and the remainder is Two.

We must always remember that in subtracting Ten we do not alter the number of the Units we have to begin with (p. 142). $21-10=?$ 11. Here we have one Unit more than two Tens, so if we subtract one Ten, we shall have one Ten and one Unit remainder. $21-10=?$ 11. $11-10=?$ 1. $1+10=?$ 11. $11+10=?$ 21.

Write Roman XXI, XXII, and Arabic 21, 22.

Once more some Multiplication.

$1\times2=2$	$1\times3=3$	$1\times4=4$	$1\times5=5$
$2\times2=4$	$2\times3=6$	$2\times4=8$	$2\times5=10$
$3\times2=6$	$3\times3=9$	$3\times4=12$	$3\times5=15$
$4\times2=8$	$4\times3=12$	$4\times4=16$	$4\times5=20$
$5\times2=10$	$5\times3=15$	$5\times4=20$	
$6\times2=12$	$6\times3=18$		
$7\times2=14$	$7\times3=21$		
$8\times2=16$			
$9\times2=18$			
$10\times2=20$			
$11\times2=22$			

$1\times 6=\ \ 6$	$1\times 7=\ \ 7$	$1\times 8=\ \ 8$
$2\times 6=12$	$2\times 7=14$	$2\times 8=16$
$3\times 6=18$	$3\times 7=21$	

$1\times 9=\ \ 9$	$1\times 10=10$	$1\times 11=11$
$2\times 9=18$	$2\times 10=20$	$2\times 11=22$

SUMS.

			1								
			2	2	3		3				
2	2	2	3	3	5	0	4	1	2	3	
0	6	6	6	9	8	1	6	5	5	2	2
5	5	3	5	1	2	9	5	8	3	7	5
4	7	6	3	4	3	7	3	6	7	6	7
9	2	4	2	3	1	4	1	2	5	4	8

$21-10.$ $21-9.$ $21-19.$ $21-20.$ $21-11.$ $21-1.$ $21-7.$ $21-17.$ $21-6.$ $21-16.$ $21-8.$ $21-18.$ $21-3.$ $21-13.$ $21-4.$ $21-14.$ $21-5.$ $21-15.$ $21-2.$ $21-12.$

$22-2.$ $22-12.$ $22-20.$ $22-10.$ $22-11.$ $22-9.$ $22-19.$ $22-4.$ $22-14.$ $22-5.$ $22-15.$ $22-6.$ $22-16.$ $22-7.$ $22-17.$ $22-8.$ $22-18.$

$21\div 7.$ $21\div 3.$

4) 21 5) 21 10) 21 9) 21

8) 21 7) 21 2) 21 6) 21 12) 21

$22 \div 11. \quad 22 \div 2.$

$$10\overline{)22} \quad 12\overline{)22} \quad 6\overline{)22} \quad 3\overline{)22}$$

$$4\overline{)22} \quad 8\overline{)22} \quad 5\overline{)22} \quad 7\overline{)22}$$

LESSON XLVII.

×. See, Tens is holding up two fingers; how many must Units hold up to-day, as we are going to have a new number? Units must hold up three fingers for Twenty-three.

And we must write Roman XXIII—Ten+Ten +three Units.

And Arabic 23—two Tens + three Units.

As Twenty-three is an odd number, we must find out whether it has any factors, or whether it is? A Prime.

Twenty-three cannot have the same factors as Twenty-two, nor as Twenty-one, we know. What are those factors? 2 and 11, and 3 and 7.

And what are the factors of Twenty? 2 and 10, and 4 and 5. So as Twenty-three is only three more than Twenty, it cannot have any of those factors.

Will Six do? $6 \times 3 = ?$ 18. Twenty-three is how many more than Twenty? Three more. And Twenty is how many more than Eigh-

teen? Two more. $3+2=?$ 5. So Twenty-three is Five more than Eighteen, and Six will not do.

What other factors has Eighteen besides Six and Three? Nine and Two. Then Nine cannot be a factor of Twenty-three.

Let us try Eight. $8\times 2=?$ 16. Twenty-three is how many more than Eighteen? Five more than Eighteen. So how many more than Sixteen is Twenty-three? Five more than Eighteen and Seven more than Sixteen. So Twenty-three is a Prime.

What is the middle number in Twenty-three? $23\div 2=?$ Eleven, and One over; so Twelve is the middle number of Twenty-three.

How can we tell that? Because when we have counted Twelve we must count Eleven more to make Twenty-three.

$23-10=?$ 13. $13-10=?$ 3. $3+10=?$ 13. $13+10=?$ 23.

That is right. We have seen before how easy it is to subtract Ten from the *teen* numbers, and to tell how many Units must be added to Ten to make a *teen* number (pp. 133, 143). The *ty* numbers are just as easy.

How many Units must we add to Twenty to make Twenty-three? The name tells us at once that we must add three Units to Twenty to make Twenty-three, for Twenty-three means? Two Tens + three Units.

And if we subtract Ten, we have just one Ten less; that is, one Ten + three Units.

To subtract Ten cannot make the number of Units either greater or less (p. 158).

Perhaps you think I have told you all this before and need not say any more about it, like a little girl who did not like it when people told her what she knew before, and used to try to stop them by saying in a great hurry, "I know lat, don't tell me lat." You see, she could not speak plainly. But if you quite understand this little riddle about adding and subtracting Tens, it will help you very much with other riddles. That is the reason why I tell you "lat" over again, that you may understand, and remember, and get quite used to it.

$20+3=?$ 23. $23-3=?$ 20. $23-10=?$ 13. $13+10=?$ 23.

Twenty-three is two Tens+three Units; so if we subtract one Ten we have one Ten and three Units left—that is Thirteen.

$23-13$? Now we have to subtract one Ten and the three Units, so only one Ten will be left.

$23-9=?$ 14. $23-14=?$ 9. $23-11=?$ 12. $23-12=?$ 11. $23-2=?$ 21. $23-4=?$ 19. That is quite easy, because we know that $23-3=20$, and $23-4$ must = One less than $23-3$, so we had to take a Unit from one of the Tens.

Roman Twenty-three — XXIII. Arabic Twenty-three—23.

What do they say? Tell me, for I am sure I need not tell you.

When you have written Twenty-three, write all the odd numbers you have learned.

LESSON XLVIII.

×. In our last lesson, when we wanted to subtract Four from Twenty-three, we found that we must take a Unit from one of the Tens.

Put twenty-three bricks on the table—two rows of Ten, and three Units in a row by themselves.

Now if you take away four bricks, you must take one from one of the rows of Ten as well as the three Units.

Now suppose you subtract six. There are the three Units, and how many more will you want to make six? Three more, so you must take those three from one of the Tens, and then what will be left? $10-3=?$ 7. So one Ten and seven Units will be left, that is $23-6=17$.

$17+6=?$ 23. Yes; if you put back the six bricks you took away, there will be just as many as there were before.

$17+6=?$ 23. Then $6+17=?$ 23. And $23-17=?$ 6. $23-8=?$ Now how many must you take from one of the Tens? Five,

because $3+5=8$. And there will be left? One Ten and five Units. $23-8=?$ 15. $15+8=?$ 23. $23-15=?$ 8. $8+15=?$ 23.

You might have found that out in another

"Which road shall we take?"

way. You might have said to yourself, $23-10=13$. Eight is Two less than Ten, so $23-8$ must be Two more than $23-10$.

$23-10=13$, so $23-8$ must $=15$.

It is a good thing to have different ways of finding out the Giant's riddles; then if the answers

are the same by each of the ways, we may feel sure we are right.

$23 - 7 = ?$ 16. $23 - 16 = ?$ 7. $7 + 16 = ?$ 23. $16 + 7 = ?$ 23. $23 - 5 = ?$ 18. $23 - 18 = ?$ 5. $18 + 5 = ?$ 23. $5 + 18 = ?$ 23. $23 - 20 = ?$ 3. $23 - 21 = ?$ 2. $2 + 21 = ?$ 23. $3 + 20 = ?$ 23.

Let us do a little sum in addition.

$1 + 2 + 4 + 14 + 2$. Let us write those figures one above the other, so—

$$\begin{array}{r} 1 \\ 2 \\ 4 \\ 14 \\ 2 \\ \hline \end{array}$$

How many Units are there? Let us see. We find that when the Units are all added together they come to Thirteen. How must we write that, do you think? 13? What does that mean? It means one Ten and three Units. But there is another Ten in our sum which must not be left out. Where are we to put that other Ten if we write *one* Ten? That will not do; we must add one Ten to the other Ten, and then we shall have two Tens.

You see, when we add many Units together they make Ten, or sometimes many Tens; then we must not write those Tens in the Units' place; that would be quite wrong. We must add the Tens to the other Tens in the sum, and write them all in the Tens' place.

A place for everything, and everything *in* its place, you know. One place for Tens, another place for Units.

Now let us look at our sum again. Thirteen Units = one Ten + three Units, so we put the three Units in *their* place, and then add the one Ten to the Ten in the sum (that is called *carrying* the Ten from the Units), and put the two Tens in *their* place.

$$\begin{array}{r} 1 \\ 2 \\ 4 \\ 14 \\ 2 \\ \hline 23 \end{array}$$

2		3	16	1		1
3	4	14	2	4	2	17
15	7	5	3	14	17	4
1	11	1	2	2	3	1
—	—	—	—	—	—	—

LESSON XLIX.

×. A new number to-day, what is it? One more than Twenty-three—that is, Twenty-four. How many fingers must Tens hold up? Two. And Units? Four. Yes, there are two Tens and four Units in Twenty-four.

So the Roman way of writing Twenty-four is? XXIV.

And the Arabic way is 24, and both Roman and Arabic tell us quite plainly what they mean.

Is Twenty-four an even number? Yes certainly, for Twenty-three is odd. How many Twos are there in Twenty-four? No doubt you are quite in a hurry to say that there must be

twelve Twos in Twenty-four, because there are eleven Twos in Twenty-two.

What are the factors of Twenty-four? Two and Twelve we know, for we have just found out that there are twelve Twos in Twenty-four, and so of course there are two Twelves (p. 71). $12 \times 2 = 24$. $2 \times 12 = 24$.

But do you remember, when we learned about factors, we found that Twelve was very rich in factors? Indeed we had really learned that before, when we talked about Twelve; only then we had not begun to call factors by their right names.

Now of course the factors of one Twelve must be also the factors of two Twelves; that is, of Twenty-four.

You know quite well that we must take six Twos to make one Twelve. Then if we want to make another Twelve we must take six more Twos. How many Twos will that be altogether? Twelve Twos.

And if we made a great many Twelves, how many Twos would there be in each Twelve? In every Twelve there are always six Twos.

How many Twos are there in Twenty-four? Twelve Twos in one Twenty-four. How many Twos in two Twenty-fours? Twenty-four Twos in two Twenty-fours; twelve for each Twenty-four.

Now I hope you can tell me the factors of

Twelve. If not, we must turn back to Lesson 41. (p. 140). The factors of Twelve are? 2, and 3, and 4, and 6.

We know how many Twos there are in Twenty-four, but can you tell how many Threes? How many Threes are there in one Twelve? Four. Then in two Twelves there must be? Two Fours of Threes—that is, eight Threes in Twenty-four. $3 \times 8 = ?$ 24.

Why here is a new factor for our number—what is it? Eight. Yes, Eight is a factor of Twenty-four. $3 \times 8 = ?$ 24. And $8 \times 3 = ?$ 24. (pp. 101, 124). *How* many Eights are there in Twenty-four? Three.

Is Eight a factor of Twelve? Oh no; in Twelve there is only one Eight, and half another, for there are only three Fours in Twelve, and Four is half Eight. You know that very well.

So in two Twelves there are two Eights and two halves of Eight, and the two halves together make another Eight, and that makes three Eights.

How many Fours are there in one Twelve? Three. And in two Twelves? Six. So $4 \times 6 = ?$ 24. And $6 \times 4 = ?$ 24 (pp. 71, 73).

Then how many Sixes are there in Twenty-four? Four. Yes, we can see that at once. If there are two Sixes in one Twelve, there *must* be four Sixes in two Twelves.

Tell me the factors of Twenty-four. 2, 3, 4, 6, 8, 12. How many factors has Twelve?—count. Four. And Twenty-four? Six.

Say all the even numbers up to Twenty-four. Now backwards. Forwards again, and then backwards, till you can say them quite quickly.

Let us count by Threes. $3 + 3 = ?$ 6. $6+3=?$ 9. $9+3=?$ 12. $12+3=?$ 15. $15 + 3 = ?$ 18. $18 + 3 = ?$ 21. $21 + 3 = ?$ 24. 3, 6, 9, 12, 15, 18, 21, 24. Forwards and backwards, again and again.

Let us count by Fours next. $4 + 4 = ?$ 8. $8+4=?$ 12. $12+4=?$ 16. $16+4=?$ 20. $20 + 4 = ?$ 24. 4, 8, 12, 16, 20, 24. Forwards and backwards, over and over again.

Now Sixes. $6+6 = ?$ 12. $12+6 = ?$ 18. $18+6 = ?$ 24. 6, 12, 18, 24. That is soon done, even if you say it six times each way.

We must not forget Eights; they will not take long. $8+8=?$ 16. $16 + 8 = ?$ 24. 8, 16, 24. Say the Eights at least six times backwards and forwards, or else perhaps poor Eight will feel jealous.

There will be so much to write in your book that we shall have to wait for another lesson till you have finished. You must not be in too great a hurry, or you will make mistakes.

Before we finish our lesson we must have the Multiplication Table with a new line—What line is that? The Twelve line.

$1\times2=2$	$1\times3=3$	$1\times4=4$	$1\times5=5$
$2\times2=4$	$2\times3=6$	$2\times4=8$	$2\times5=10$
$3\times2=6$	$3\times3=9$	$3\times4=12$	$3\times5=15$
$4\times2=8$	$4\times3=12$	$4\times4=16$	$4\times5=20$
$5\times2=10$	$5\times3=15$	$5\times4=20$	
$6\times2=12$	$6\times3=18$	$6\times4=24$	
$7\times2=14$	$7\times3=21$		
$8\times2=16$	$8\times3=24$		
$9\times2=18$			
$10\times2=20$			
$11\times2=22$			
$12\times2=24$			

$1\times6=6$	$1\times7=7$	$1\times8=8$	$1\times9=9$
$2\times6=12$	$2\times7=14$	$2\times8=16$	$2\times9=18$
$3\times6=18$	$3\times7=21$	$3\times8=24$	
$4\times6=24$			

$1\times10=10$	$1\times11=11$	$1\times12=12$
$2\times10=20$	$2\times11=22$	$2\times12=24$

Exercises.

1. Write all the numbers up to Twenty-four in Roman figures, not forgetting to mark the Primes.
2. Write all the numbers up to Twenty-four in Arabic figures, marking the Primes.
3. Write all the even numbers up to Twenty-four.

4. Begin with Three, and go on by Threes up to Twenty-four.

5. Do the same with Four. Then with Six. With Eight. With Twelve.

SUMS.

		11			10
2		7	10		5
6	12	2	8	13	4
8	4	3	3	6	2
7	8	1	2	2	1
—	—	—	—	—	—

$24 \div 6$. $24 \div 2$. $24 \div 8$. $24 \div 12$. $24 \div 3$. $24 \div 4$. $24 \div 24$.

LESSON L.

IN our last lesson we finished the Two line as far as it is generally learned. If you would like to make it longer you can go on with it as much further as you choose.

But whatever you do about the Two line, you must certainly go on with the other lines till each of them comes to Twelve. How is that to be managed? Must we still go on making a lesson of each number separately? I think we need not stop to do that, now that you have found out so many of the Giant's little riddles, and understand quite well the meaning of the names of the numbers.

What does Twenty-four mean? Two Tens + four Units. And what comes next? Twenty-five. And next? Twenty-six. Twenty-seven. Twenty-eight. Twenty-nine. You can tell what is meant by the name of each one of those numbers. I am sure you must know it so well that I shall not write it.

Now let us see. What was the last number we found out in the Three line? $8 \times 3 = ?$ 24. How many more than 8×3 is 9×3? Three more, because if you have three Eights of bricks, or anything else, you must add One to each of the three Eights to make it into a Nine. So $9 \times 3 = 24 + 3$, and that $= ?$ 27. $4 + 3 = ?$ 7. And $24 + 3 = 27$. 9×3 then $= ?$ 27. And $3 \times 9 = ?$ 27.

$24 \div 4 = ?$ 6. $6 \times 4 = ?$ 24. How many must we add to 6×4 to make 7×4? We must add Four, because if we have four Sixes, we must add One to each of the Sixes to make four Sevens; that is, we must add four Ones. $24 + 4 = ?$ 28. $7 \times 4 = ?$ 28. And $4 \times 7 = ?$ 28.

$4 \times 5 = ?$ 20. $20 + 5 = ?$ 25. Then, $5 \times 5 = ?$ 25.

Now do you see how to finish making the Multiplication Table? Every day you must try to multiply a number—the one next to that you did last time; and when you have done this you will generally find that you know how to multiply another number as well, because you know

that in multiplying it does not matter which factor you take first. 8×3 and 3×8 both $= 24$ (pp. 71, 73).

So that you will generally be able to make two lines in your Multiplication Table, and it will grow a little almost every day; and then you must be sure to say all that you know aloud, so that you may get to know it very well indeed, and always be ready to use it when you want to find out other riddles.

When the factors of any number are both the same, as in 5×5, only one line will grow, as I need hardly tell you; but if you go on little by little, you will presently find you have made the whole Multiplication Table for yourself.

I dare say that somewhere in your house there may be a book which has all the Multiplication Table in it, so that people may learn it if they like. Suppose you were to copy this, would that be a good plan do you think? What does Giant Arithmos want people to do? To *find out* his riddles themselves, not to ask some one else to find them out for them, or to copy the answers out of a book. People who do such things are sure to be disappointed when they try to find out the Giant's harder riddles. Then they call him a cross old thing, and all sorts of hard names, which is not fair when they have not been willing to take the trouble of doing things properly. Giant Arithmos will not help such people much.

So you see you must find out the Multiplication Table for yourself. It will not be hard to do.

1+4+5+6+7+2=? 25. 2+3+4+5+6+7+2=? 29. 3+3+4+5+6+2=? 23. 4+3+4+5+6+7=? 29. 5+3+4+5+6+5=? 28. 6+2+3+4+7+6=? 28. 9+6=? 15. 7+6=? 13. 19+9=? 28. 17+10=? 27. 19+10=? 29. 14+11=? 25. 15+9=? 24. 13+12=? 25. 7+8=? 15.

SUMS.

10		10		13	9
13	12	11	14	2	10
2	9	4	10	7	7
1	7	3	5	6	1

25÷5. 20÷5. 28÷7. 28÷4. 15÷5. 15÷3. 21÷3. 21÷7. 22÷11. 18÷6. 18÷3. 16÷8. 16÷4. 16÷2.

LESSON LI.

×. 24 − 19 = ? That is very easy, for we know that 24 − 20 = 4. If we subtract both the Tens, only the Units will remain; and if we leave one Unit out of one of the Tens, and add it to the four Units, we shall have Five remainder (p. 142).

Now we will write that sum in another way—

$$\begin{array}{r} 24 \\ 19 \\ \hline \end{array}$$

We must be careful, when we write Subtraction sums in that way, to put the larger number at the top (p. 35), and to put the Units under the Units, the Tens under the Tens.

Let us begin with the Units, and subtract 9 from 4. Can we do it? No, because 9 is more than 4.

What is to be done? Take some from one of the Tens? That is quite right. We will take one of the Tens from the Twenty, borrow Ten, as it is called, and add it to the four Units. $10+4=?$ 14.

Now we can subtract 9. $14-9=?$ 5. Five what? Five Units. Then where must we write the Five? In the Units place, of course.

$$\begin{array}{r} 24 \\ 19 \\ \hline 5 \end{array}$$

But what is to be done with the Ten which we borrowed? If we put it back to the Twenty, and said $2-1$, and wrote the 1 in the Tens' place, we should say that $24-19=15$, and that would be wrong.

Besides, we have subtracted some of the Units from that Ten, so that it is no longer Ten.

Then if we have taken one Ten from the two Tens, how many are left? Only one Ten. And $1 - 1 = ?$ 0.

Generally however, when people have borrowed Ten in subtracting, they pay it back to the lower figure. If we add one Ten to the one Ten in 19, we shall have two Tens.

Subtract those two Tens from the two Tens in 24. $2 - 2 = ?$ 0.

You see the remainder is the same whether we take One from the larger number of Tens before subtracting the smaller, or add One to the smaller before subtracting it from the larger number.

But we must take great care not to use the same Ten twice over; and, when we have made a Ten into Units, and subtracted some of them, not to use it again as a whole Ten.

So perhaps it is best to follow the general plan, and when we borrow Ten from the larger number always pay it back to the smaller.

$$\begin{array}{r} 29 \\ 17 \\ \hline 12 \end{array}$$

That is a very easy sum. $9 - 7 = ?$ 2. Two what? Units. $2 - 1 = ?$ 1. One what? One Ten. So, $29 - 17 = ?$ 12.

Of course you could have done that without writing it, for you know that $29-10=?$ 19. And as you had also to subtract seven Units, you would say $19-7=?$ 12.

Or you might have said—If I subtract the 9 Units 20 will remain. From one of those Tens I must subtract Units enough to make 17 when added to the 9 Units. How many more than 9 is 17? 8 more, $9+8=?$ 17. Then 8 must be subtracted from 20. $20-8=?$ 12.

$24-18$. You know that 24 and 18 have one factor the same—what is it? (pp. 145, 168). 6. How many Sixes are there in 24? 4. And in 18? 3. Then if we subtract three Sixes from four Sixes, what will the remainder be? One Six. $24-18=?$ 6. $24-6=?$ 18. $24-12=?$ 12.

$24-4=?$ 20. $24-20=?$ 4. $24-8=?$ 16. $24-16=?$ 8.

$24-21=?$ 3. $24-10=?$ 14. $24-11=?$ 13. $24-9=?$ 15. $24-15=?$ 9. $24-13=?$ 11.

If the last two questions seem rather hard, they are not really so. You can either do them in the ways I have shown you before, or you can think how many must be added to 15 or to 13 to make 24, and then you will know how many will be left if 15 or 13 be subtracted from 24. $15+10=?$ 25. So 9 must be added to 15 to make 24, and 9 will be the remainder if we subtract 15 from 24 (p. 73).

We need not have much trouble with 13. We can say, Oh Mr. Thirteen, we know that you are just one more than Twelve, and we know that $12+12=24$, so of course $13+11$ will $= 24$. And $24-13$ must $=11$.

Besides, we know that $24-12=12$, and in that way we see at once that $24-13$ must $= 11$.

$25-10=?$ 15. $25-15=?$ 10. $25-11=?$ 14. $25-14=?$ 11. $25-5=?$ 20. $25-20=?$ 5. $25-16=?$ 9. $25-9=?$ 16. $25-7=?$ 18. $25-18=?$ 7. $25-8=?$ 17. $25-17=?$ 8. $25-19=?$ 6. $25-6=?$ 19.

$28-18=?$ 10. $28-10=?$ 18. $28-19=?$ 9. $28-9=?$ 19. $28-11=?$ 17. $28-17=?$ 11. $28-7=?$ 21. $28-21=?$ 7.

$28-14=?$ 14.

What are the factors of 28? 7 and 4. And of 14? 7 and 2. Then if we subtract two Sevens from four Sevens what will remain? Two Sevens. So $28-14=?$ 14. And $14\times2=?$ 28. $2\times14=?$ 28. $28-15=?$ 13. $28-13=?$ 15.

Sums.

$25\div5$. $28\div7$. $28\div4$.

Subtraction—

28	26	29	28	27	26
17	13	14	19	18	19

25	24	27
17	18	19

Write the numbers from Twenty-five to Twenty-nine in Roman and in Arabic figures.

LESSON LII.

×. LET us play again at our game of Tens and Units, and hold up our fingers for 24. Now for 25, 26, 27, 28, 29.

How many fingers does Tens hold up? Two. Because? Because each of these numbers is two Tens + some other number.

How many fingers does Units hold up for 29? Nine. Because? Because 29 = 2 Tens + 9 Units.

If we add 1 to 29, how many fingers must Units hold up? None at all, for 29 + 1 will equal another Ten. So Tens must hold up? Three fingers, for we shall have three Tens.

What do we call three Tens. Threety? Not exactly. Did we say Threeteen? No, we said *Thir*teen. And we call three Tens *Thir*ty. What is the difference between Thirteen and Thirty? Thirteen means? 10 + 3. And Thirty means? 3 × 10 (p. 152).

How shall we write Thirty? I believe you can tell. Roman, XXX. Arabic, 30. Both figures tell us quite plainly what they mean.

Now let us count again, and Tens and Units

must hold up their fingers. Thirty-one, Thirty-two, Thirty-three, Thirty-four, Thirty-five, Thirty-six, Thirty-seven, Thirty-eight, Thirty-nine.

What next? Next will come another Ten. How many Tens will that make? Four Tens.

What do we call four Tens? Forty. That is right; but I must tell you that when we spell Forty we leave out the *u* in Four, and spell the word f-o-r-t-y.

Can you write Forty in figures? Arabic Forty, of course, you can,—40. But what were we obliged to do when we wanted to write the Roman figures for four Units? (p. 66). We had to wait till we knew how to write Five. So now that we want to write four Tens, we must wait till we know how to write five Tens.

So we had better begin to count again. This time you shall say the numbers till we come to another Ten. What do we call five Tens? Fifty.

How shall we write five Tens?—Arabic, of course, 50. We just have to put a Five in the Tens' place, and a Nought in the Units' place. That is soon done.

But Roman Fifty? Lay your left hand flat on a piece of paper. Keep the four fingers close together, and stretch out the thumb as far as it will go. Take a pencil and draw it along by the first finger and thumb. Lift up your hand. You will find a figure something like this—L. That is the Roman figure for Fifty.

Now how shall we write Forty? Put One before it to show that it is less than Fifty? One what, one Unit? Oh no; Forty is Ten less than Fifty; we must put one Ten before the Fifty. That is right; here is the Roman Forty —XL.

Count again. The next Ten will be? Six Tens. And its name? Sixty. Arabic? 60. Roman? How did we write six Units? Put One after the Five. Why? To show that we meant Five+One.

Now let us write Sixty. You see we must put a Ten after the Fifty to show that we mean Fifty+Ten. LX.

Count another Ten. How many Tens have we now? Seven. What do we call seven Tens? Seventy. And of course this is how we write Seventy:—70. LXX.

Count again till we come to eight Tens. Eighty. 80. LXXX.

Once more, and we shall have counted nine Tens. Ninety. 90. But how must we write the Roman Ninety? When we have counted one more Ten, how many will that make? One more Ten will make ten Tens.

When we had counted nine Units you remember we had to wait to write Roman Nine till we knew how to write Ten (p. 103). What must we do now, then? Wait to write Ninety till we know how to write ten Tens? Yes, that is what

we must do; but I think you have learned enough new figures to-day, so Roman Ninety must wait till another lesson, perhaps till the next lesson but one.

Before we leave off, I want you to notice how kindly these *ty* numbers help us with our Multiplication. Thirty says, "I am 3×10. I *could* not be anything else. No one could possibly be so silly as to fancy I could be 5×10 or 8×10, or any other number multiplied by Ten; and every one knows, or at least every one *ought* to know, that the end of my name shows that I must be *some* number multiplied by Ten. Yes indeed, we *ty* numbers are much bigger people than the Units, or the *teen* numbers either, I can tell you." Dear, dear, how conceited some of the numbers are, and after all there are much bigger numbers than *ty* numbers, as we shall be able to show this boasting Mr. Thirty before very long.

EXERCISE.

Write all the *ty* numbers except Ninety in Roman and Arabic figures, and write the meaning of their names in this way:—XX, $20 = 2 \times 10$.

LESSON LIII.

$\times$. $3 \times 10 = ?$ 30. $10 \times 3 = ?$ 30. And so on with each of the other Tens. Do you see

that the name of one of these *ty* numbers tells us at once two of the factors of the number?

What number is it which is always a factor of these numbers? Ten. Because? Because *ty* means that some number is multiplied by Ten. And the first part of the name tells us *what* number it is that is multiplied by Ten.

What are the factors of 20? 2 and 10. What else? 4 and 5. There are two Fives in every Ten, so there must be four Fives in two Tens.

What are the factors of 30? 3 and 10. And? And 5. How many Fives are there in 30? Two Fives in each Ten, so in three Tens there must be six Fives. Yes, $5 \times 6 = ?$ 30. And $6 \times 5 = ?$ 30.

Why here is a new factor for 30—what is it? 6. How many factors has 30 then? 3 and 10, and 5 and 6; that makes four factors for 30.

But there is one more which we must not forget. What are the factors of 10? 5 and 2. Then 2 must be a factor of 30. How many Twos in 10? 5. How many Twos in three Tens? $3 \times 5 = ?$ 15. Fifteen Twos in three Tens. $2 \times 15 = ?$ 30. $15 \times 2 = ?$ 30 (p. 71). Now let us say the factors of 30 again. 2 and 15, and 5 and 6, and 10 and 3.

What are the factors of 40? 4 and 10. And? And 5 and 2. How many Fives in four Tens? Eight Fives in four Tens. $5 \times 8 = ?$ 40. And $8 \times 5 = ?$ 40. So 8 is a factor of 40.

And something else besides—what can that be? Has Four any factors? Four has one factor —that is, Two, for $2 \times 2 = 4$. Then 2 must be a factor of 40. How many Twos are there in 40? $4 \times 10 = ?$ 40. So $2 \times 20 = ?$ 40. And $20 \times 2 = ?$ 40 (p. 71). Now say all the factors of 40. 2 and 20, and 5 and 8, and 10 and 4. Six factors for 40.

The factors of 50? 5 and 10. $5 \times 10 = ?$ 50. $10 \times 5 = ?$ 50.

Has Five any factors? No; Five is a prime and has no factors (p. 105). But Ten has factors —what are they? 2 and 5. So if 2 is a factor of one Ten, it must also be a factor of five Tens, or 50.

How many Twos are there in Ten? Five Twos in one Ten. So in five Tens there must be five Fives of Twos, one Five of Twos for each Ten. $5 \times 5 = ?$ 25. $2 \times 25 = ?$ 50. And $25 \times 2 = ?$ 50.

The factors of 50 are then? 2 and 25, and 10 and 5.

Now we will have the factors of 60—What are they? 6, and 10, and 5. Yes, $6 \times 10 = ?$ 60. $10 \times 6 = ?$ 60.

And $5 \times$ —— By what must 5 be multiplied to make 60? By 12, for if there are 6 Tens in 60, there must be 12 Fives, two Fives in each Ten. $5 \times 12 = ?$ 60. $12 \times 5 = ?$ 60.

What is the other factor of Ten? Two. So

Two must always be a factor of the *ty* numbers as well as Five, because all the *ty* numbers have Ten for a factor, and Two we know is a factor of Ten.

In one Ten how many Twos? Five. Then how many Fives of Twos in six Tens? Six Fives of Twos in six Tens. $5 \times 6 = ?$ 30.

So there are 30 Twos in Sixty. $2 \times 30 = ?$ 60. $30 \times 2 = ?$ 60.

$12 \times 5 = ?$ 60. What are the factors of 12? 2, 3, 4, 6.

How many Twos are there in 60? 30. And how many Threes? Well, we know how many Sixes there are in 60; the name tells us that there are ten. Then if there are ten Sixes there must be, how many Threes? (p. 90). 20. So there are. $3 \times 20 = ?$ 60. $20 \times 3 = ?$ 60.

Besides, we could have told it in another way. We have found that there are two Thirties in 60, and we know how many Threes there are in each 30? 10 Threes in each Thirty; 20 Threes in two Thirties. Any one who has found out as many of the Giant's riddles as you have can tell that quite easily.

Now we must find out how many Fours there are in 60. How many are there in 12? 3. And how many Twelves are there in 60? 5. Then how many Fours are there in five Twelves? $3 \times 5 = ?$ 15. 15 Fours in five Twelves. $4 \times 15 = ?$ 60. $15 \times 4 = ?$ 60.

We know how many Sixes there are in 60, so now we can say all the factors of 60. 2 and 30, and 4 and 15, and 6 and 10, and 12 and 5, and 20 and 3. What great factors! That is because we are beginning to talk about great numbers. We are getting on.

Next we must have the factors of 70—What are they? 7 and 10, and 5 and 2.

Has 7 any factors? No; 7 is a? A prime.

How many Sevens in 70? 10. $7 \times 10 = ?$ 70.

How many Tens in 70? 7. $10 \times 7 = ?$ 70.

How many Fives? If there are 7 Tens there must be? 14 Fives. $5 \times 14 = ?$ 70. $14 \times 5 = ?$ 70 (p. 71).

So 14 is a? A factor of 70. How many Twos in 70? 5 for each 10. 7 Tens in 70. $5 \times 7 = ?$ 35. So there are 35 Twos in 70. $2 \times 35 = ?$ 70. And $35 \times 2 = ?$ 70. Now then, what are the factors of 70? 2 and 35, and 7 and 10, and 14 and 5.

What are the factors of 80? 8 and 10, and 5 and 2 (p. 185).

And something else you can see at once, for what are the factors of 8? 4 and 2. Then 4 must be a factor of 80.

How many Eights in 80? 10. $8 \times 10 = ?$ 80. Tens? 8. $10 \times 8 = ?$ 80. Fours? 10 Twos of Fours; that is, two Fours for each Eight. 20 Fours. $4 \times 20 = ?$ 80. $20 \times 4 = ?$ 80.

How many Fives? 8 Twos of Fives, because there are eight Tens in Eighty. 16 Fives. $5 \times 16 = ?$ 80. $16 \times 5 = ?$ 80.

And how many Twos? 8 Fives of Twos. $5 \times 8 = ?$ 40. 40 Twos. $2 \times 40 = ?$ 80. $40 \times 2 = ?$ 80.

The factors of one more number — what number do you think? 90. That is right. What are the factors of 90? 9 and 10, and 5 and 2.

What else? What factor has 9? 3, for $3 \times 3 = ?$ 9. So 3 is? A factor of 90.

How many Nines in 90? 10. $9 \times 10 = ?$ 90.

And Tens? 9. $10 \times 9 = ?$ 90.

How many Fives. There are 9 Tens, so there must be? Twice 9 Fives, 18 Fives. $5 \times 18 = ?$ 90. $18 \times 5 = ?$ 90.

How many Threes? 3 for each 9.

And how many Nines? 10. So there are 30 Threes in 90. $3 \times 30 = ?$ 90. $30 \times 3 = ?$ 90.

So 30 is a factor of 90, and what are the factors of 30? 2 and 15, and 5 and 6, and 10 and 3.

How many Fifteens are there in each 30? 2 Fifteens in each 30.

Then in 3 Thirties there must be? 6 Fifteens. $15 \times 6 = 90$. $6 \times 15 = ?$ 90. Why here is *another* factor for 90—What is it? 6.

How many Sixes are there in 90? 15.

How many Twos? 5×9 Twos, 5 for each 10. $5 \times 9 = 45$, so there are 45 Twos in 90. $2 \times 45 =$? 90. $45 \times 2 =$? 90.

Then the factors of 90 are? 2 and 45, and 3 and 30, and 5 and 18, and 6 and 15, and 9 and 10.

SUMS.

$20 \div 2$. $30 \div 2$. $20 \div 4$. $20 \div 10$. $30 \div 10$. $30 \div 3$. $20 \div 5$. $40 \div 10$. $90 \div 10$. $50 \div 10$. $50 \div 5$. $50 \div 25$. $50 \div 2$. $30 \div 6$. $30 \div 5$. $30 \div 10$. $40 \div 2$. $40 \div 20$. $40 \div 8$. $40 \div 5$. $60 \div 2$. $60 \div 30$. $60 \div 10$. $60 \div 5$. $60 \div 15$. $60 \div 4$. $30 \div 15$. $60 \div 12$. $70 \div 10$. $70 \div 5$. $70 \div 14$. $70 \div 35$. $70 \div 2$. $80 \div 10$. $80 \div 20$. $80 \div 40$. $80 \div 8$. $80 \div 4$. $80 \div 2$. $80 \div 5$. $80 \div 16$. $90 \div 9$. $90 \div 10$. $90 \div 5$ $90 \div 18$. $90 \div 3$. $90 \div 2$. $90 \div 30$. $90 \div 6$.

LESSON LIV.

×. What is the next Ten to Ninety? Tenty? That would not do very well, for we should always be making mistakes between Tenty and Twenty. Besides, we call ten Units a Ten, and all the numbers which can be divided by Ten without any remainder (p. 149) we call Tens.

So don't you think we ought to have a new name for 10 Tens, and rather a grand name too, because 10 Tens is such a great number? We

call 10 Tens a Hundred. $10 \times 10 = ?$ One Hundred.

Now, as Roman Ninety has been waiting all this time, perhaps we had better find out how to write one Hundred before we find out what are its factors. How *shall* we write it ?

The Romans wrote one Hundred thus—C.

The Latin word for hundred is *Centum,* and C, as you see, is the first letter in *Centum.* Do you know any English word which comes from *Centum ?* There are several; I will tell you one. Century means a hundred years.

Now let us write Ninety. How must we do it ? Put X before the C to show that Ninety is Ten less than a Hundred ? That is right. XC is the way the Romans wrote Ninety.

But how shall we write Arabic Hundred. We have used up all the figures. Must we have a new one ? How did we manage when we had used up all the figures in writing 9 Units ? Did we make a new figure for the Ten ? (p. 113). Oh no, we made a new place for the figures, the Tens' place.

So now we must make ? A Hundreds' place.

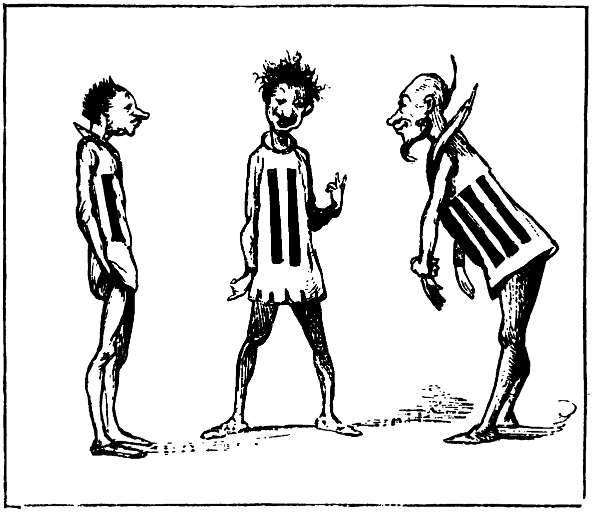
Let us try. We want to write one Hundred. Any Units ? No, not any Units. Then what must we put in the Units' place ? 0, to show that there are none.

Any Tens ? No. So we must put ? 0 in the Tens' place. 00.

Any Hundreds? Yes, one. Well, we will make a place for the Hundreds next to the Tens, so—100.

There is the Hundreds' place in the third row.

1. What does that mean? One Unit.



Units, Tens, Hundreds.

10. What does the 1 mean there? One Ten.

How is it that the second 1 means so much more than the first? Because we have changed its place and put it in the second row. $1 \times 10 = ?$ 10.

So the second 1 is worth ten times as much as the first.

100. What does the 1 mean there? One Hundred. $10 \times 10 = ?$ 100.

So the third 1 is worth ten times as much as the second, and the second ten times as much as the first.

By putting 0 after the 1 we multiplied the 1 ten times. And it would be just the same with any other number. By putting a 0 after it we should multiply it by Ten, for we should make it worth ten times as much as it was before.

If there were several figures in the number, and we put a 0 at the end, each figure would be worth? Ten times as much as it was before.

Let us try 3. That means? 3 Units.

30 means? 3 Tens, which we call Thirty. Ah, here is that conceited Thirty again.

300. What does that mean? That means three Hundred; and three Hundred = ten times Thirty. $30 \times 10 = ?$ 300. $10 \times 30 = 300$.

Is not that an easy way of multiplying by Ten, and finding out some of the Giant's riddles? Thirty need not have been so boastful, for Three Hundred is worth ten times as much. It is not very wise of numbers, or of people either, to think too much of themselves; they are sure to find other numbers or other people much more important than they are.

Now what are the factors of 100? 10, we know, and so, of course, 5 and 2 must also be factors of 100 (p. 185). $100 \div 10 = ?$ 10.

Ten Tens in 100. And how many Fives in ten Tens? Two for each Ten, makes twenty Fives. $5 \times 20 = ?$ 100. $20 \times 5 = ?$ 100. So 20 is a factor of 100. And how many Twos? There must be a great many Twos in 100 if there are 20 Fives. How many Twos are there in each Ten? 5. Then in 10 Tens there are? $5 \times 10 = ?$ 50. There are 50 Twos in one Hundred. What are the factors of 20? (p. 152). We have found out how many of each of those factors there are in 100, except 4. How many Fours are there in 100? How many are there in 20? $20 \div 4 = ?$ 5. Then as there are 5 Twenties in 100, there must be? 25 Fours, 5 Fours for each 20, and $5 \times 5 = ?$ 25. $4 \times 25 = ?$ 100. $25 \times 4 = ?$ 100.

The factors of 100 are then? 10, and 2 and 50, and 4 and 25, and 5 and 20.

EXERCISES.

1. Write Roman one Hundred, and Arabic one Hundred.
2. Write all the factors of 100.
3. Write the Hundreds up to nine Hundred in Arabic figures.

SUMS.

$90+10$. $98+2$. $96+3$. $96+5$. $92+9$. $97+3$.

$100 \div 5$. $100 \div 50$. $100 \div 2$. $100 \div 4$. $100 \div 10$. $100 \div 25$.

LESSON LV.

x. 1+2=? 3. 3 what? 3 Units.

Yes, and one Ten+2 Tens=? 3 Tens.

And 1 Hundred+2 Hundred=? 3 Hundred.

10+20=? 30. 3+4=? 7. 30+40=? 70. 300+400=? 700. 5+4=? 9. 50+40=? 90. 500+400=? 900. 2+6=? 8. 20+60=? 80. 200+600=? 800. 3+6=? 9. 30+60=? 90. 300+600=? 900.

10+12=? 22. We do not change the Units when we add Ten, or any number of Tens, as you know (p. 143). 22+10=? 32. 32+10=? 42. 42+10=? 52. 56+10=? 66. 10+36=? 46. 36+20=? 56. 77+20=? 97. 77+30=? 107. 30+70=? 100 50+54=? 104. 90+20=? 110.

7+3=? 10. 70+30=? 100. 77+30=? 107. 77+33=? Ah, now we have to change the Units, for we have to add Units as well as Tens.

Let us begin with the Tens. 70+30=? 100. Now the Units. 7+3=? 10. Why, that is another Ten to be added to the Hundred. One Hundred and one Ten, and how many Units? None at all; they have been added together, and have made a Ten. So—77+33=? 110.

77+34=? One more than another Ten, one Unit more. 77+34=? 111.

When we have to add Units, of course that

changes the Units. The best way is to add the Hundreds first, if there are any, then the Tens, then the Units; that is, it is best to begin with the highest numbers when we want to add them without writing them. When we *write* an Addition sum we know we begin with the Units (p. 165).

27+34. Begin with the Tens. 20+30=? 50. Now the Units. 7+4=? 11. What must we do with that Ten? Add it to the 50. 50+10=? 60. But we must not forget the Unit; you know there is one Unit in 11. 27+34=? 61.

48+37=? The Tens = seven Tens, that is 70, and the Units = fifteen Units, so 48+37=? 85.

54+29=? 83. 28+36=? 64. 29+13 =? 42. 66+27=? 93. 25+45=? 70. 201+302=? 503. 403+209=? 612. 290+310=? 600. 290+320=? 610.

Can we play our old game of Units and Tens now? (p. 137). Yes; but we must either get some one to be Hundreds, or we must take ten bricks and lay them flat on the table to do instead of ten fingers. Then when the fingers ought to be held up we can set up the bricks. Perhaps, as you have been Units so long, you would like to be Hundreds now, for a change; then the bricks can be Units. When we have held up our fingers for a number, you may write it on a piece of paper. You can easily do that.

One hundred. One hundred and one. One hundred and ten. Two hundred and twelve. Two

hundred and twenty. Three hundred and thirty-three. Seven hundred and fourteen. Eight hundred and forty. Nine hundred and ninety-nine. Six hundred and fifty-four. Four hundred.

I do not think it is fair for me to say all the numbers. You must have your turn too. Say any numbers you like, and we will try to hold up the right number of fingers. Only do not say any number higher than 999, because, you know, we have not talked about ten hundred yet.

Now let us do a sum in addition. I will say the numbers, and we will hold up our fingers, and then you shall write the numbers one under the other. You must be careful to write them properly (p. 165). If you were to put the Units under the Tens, what would happen? Then we should add the Units to the Tens, and that would be a great mistake, because the Tens are worth ten times as much as the Units. Would not Thirty be angry if he were added to a Unit?

Ten. Forty-nine. Eighteen. Three hundred and forty-five. Four hundred and sixty-seven. Eleven.

$$\begin{array}{r} 10 \\ 49 \\ 18 \\ 345 \\ 467 \\ 11 \\ \hline 900 \end{array}$$

There is a fine large sum. The first row, the Units, comes to Thirty—that is, three Tens, and no Units over; so we must put down 0 in the Units' place, and carry the three Tens to the next row, the Tens' row. The next row comes to Twenty, and twenty Tens we know = 200 (p. 191) so we have to put 0 in the Tens' place also, and carry Two to the Hundreds' row, which comes to Nine; so the answer to our sum is 900.

900 − 456. You might say to yourself 900 − 400 = ? 500. 500 − 50 = ? 450. 450 − 6 = 444. So 900 − 456 = ? 444. But sometimes we want to write such a sum. You have learned how to do it (p. 175).

$$\begin{array}{r} 900 \\ 456 \\ \hline 444 \end{array}$$

0 − 6. How must we manage that? Borrow 10, and say 10 − 6 = ? 4.

Then we must pay back the Ten, and say 0 − 6 in the Ten line.

What must we do next? Borrow Ten from the next line. That is right. The next line is the Hundreds' line; so, if we borrow one Hundred, that will be equal to ten Tens, and instead of saying 0 − 6, we can say 10 − 6 = ? 4.

Then we must pay back the One we borrowed, and say 9 − 5 = ? 4.

999 ÷ 3. Have you forgotten how we used

to write our division sums? (p. 51). Let us write this in the same way.

$$3\underline{)\ 999\ }$$
$$333$$

First we say how many Threes in Nine? We know there are three Threes in Nine, so we write 3 under the first 9.

What does the first 9 mean? 900. Surely there are more than 3 Threes in 900. Yes; if there are 3 Threes in Nine, there must be 3 Tens of Threes in 9 Tens, and 3 Hundreds of Threes in 9 Hundred. So we put this 3 in the Hundreds' place, and then it will mean? 300.

Then we go on to the next figure, which means? Nine Tens. And say? Threes in Nine = 3. We write that 3 in the Tens' place, and go on to the Units, and say? $9 \div 3 = 3$. That 3 means? 3 Units.

So we find that $999 \div 3 = ?$ 333.

Here is another sum. $126 \div 3$. $3\underline{)\ 126\ }$

$$42$$

$1 \div 3$. Can we do that? No; One cannot be divided by Three. What must we do? Borrow One? When we borrowed One in Subtraction we began with the Units, but in Division we begin with? The Hundreds. Yes, or with whatever may be the highest number. It would be of no use to borrow a Ten to help us to divide the Hundred.

Instead of that we had better make the Hundred into Tens; how many Tens shall we have then? Ten Tens.

And how many Tens are there in the Tens' place? Two. Then altogether there are? 10 Tens + 2 Tens = ? 12 Tens.

We can divide 12 by 3. $12 \div 3 = ?$ 4. Then there are 6 Units. $6 \div 3 = ?$ 2. And so the sum is done.

I wanted you to understand exactly how that sum was done. That is the reason why I asked you how many Tens there are in a Hundred. But generally, you need not stop to say to yourself how many Tens are there in this Hundred or in these Hundreds. You know that in a row of figures the first figure in the row is worth ten times as much as it would be if it stood in the next place. It is like a Ten to the next row.

So we can take the first two numbers together, and read them as if one were Tens and the other Units. In our sum we said $12 \div 3 = 4$. 12 Units $\div$ 3 = 4 Units, and 12 Tens $\div$ 3 = 4 Tens.

Only we must write the 4 in the right place, under the number we are dividing.

Here is another sum.

$$3\underline{)\ 225}$$
$$\ \ 75$$

We say $2 \div 3$; we cannot; so we must take the next figure and say $22 \div 3 = 7$, and 1 over. That one over is a Ten, so we add it to the Five, and say $15 \div 3 = 5$.

If the sum had been $226 \div 3$, there would have been one Unit over, for 226 cannot be divided by 3 without a remainder (p. 149).

A long time ago we learned how to write that One over, so long that I dare say you have forgotten, and we must turn back to the old lesson (p. 63).

Here is a multiplication sum. 395×2.

We must write the number to be multiplied, and then put the number by which it is to be multiplied underneath.

Under which figure must we put that number; had we better begin with the Hundreds or with the Units? If we begin with the Hundreds we may find that the Tens will make another Hundred, which will be awkward, so it will be better to begin with the Units, and write our sum in this way—

$$\begin{array}{r} 395 \\ 2 \\ \hline 790 \end{array}$$

$5 \times 2 = ?$ 10. We put the 0 in the Units' place.

What must we do with the Ten? Carry it to the other Tens? Yes, that is what we must do after we have multiplied them. $9 \times 2 = ?$ 18. Add the Ten from the Units. $18 + 1 = ?$ 19.

Put down 9 and carry the 1 to the Hundreds, for $10 \times 10 = ?$ 100. $3 \times 2 = ?$ 6. $6 + 1 = ?$ 7.

SUMS.

Twelve + Twenty-four + Thirty-six + One hundred and two + Three hundred and forty-seven + Four + One + Ninety-nine + One hundred and fifty-four.

Nine hundred and ninety-nine − Eight hundred and eighty. Eight hundred and eighty − Four hundred and ninety-nine.

Seven hundred and seven ÷ Seven. Six hundred and fifty-four ÷ Six.

One hundred and fifty × Four. One hundred and ninety × Five.

LESSON LVI.

×. It was very easy to write the Hundreds in the Arabic way; you had only to put the number of the Hundred you wished to write in the proper place, and the thing was done. It is not difficult either to write the Hundreds in the Roman way.

How did the Romans write 100? C. 200—CC. 300—CCC. 400—CCCC.

You see they wrote 400 without having anything to do with 500; but they had a different figure for 500; this is it—D. I do not know why they chose D for 500, unless it were that it is next to C in the alphabet. At any rate we can

remember in that way what is the Roman figure for 500.

Now I think you can tell how to write the other Hundreds. 600 — DC. 700 — DCC. 800—DCCC. 900—DCCCC.

What is the next Hundred to 900? Ten hundred. So it is; but ten Tens have a name to themselves, so ten Hundreds will expect to have a name too, and we call ten Hundred one Thousand. $100 \times 10 = ?$ One Thousand. $10 \times 100 = ?$ One Thousand.

The Romans wrote One Thousand in this way —M, and I can tell you why they did so. Because M is the first letter of *Mille*, which is the Latin (p. 24) word for a Thousand.

Now for the Arabic way of writing a Thousand; how is that to be done? Make a? A new place for it. Certainly. Tens has a place, and Hundreds has a place, so Thousands must have a place too. Thousands' place is the fourth place—thus, 1000.

I hope you will always remember that, and never call any figure which does not stand in the fourth place a Thousand.

1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000. What next? Ten Thousand.

Ten Thousand does not have a new name; we just call it Tens of Thousands.

When we speak of Tens we mean? Tens of Units. When we have two Tens of Units we

call the number? Twenty. When we speak of Ten Thousand we *mean* Ten Thousand, and if we have two Tens of Thousands we call the number? Twenty Thousand.

Tens of Thousands can do without a new name, but it must have a new place in the Arabic figures, for if we wrote it in one of the old places what would happen? It would not mean Tens of Thousands at all, but something else quite different.

Tens of Thousands has the fifth place—10,000. You see, if we were reading the Arabic figures in a hurry, we might make a mistake as to the number of Noughts, so, in order to prevent this, when we are obliged to write more than three Noughts, it is a good plan to put a comma after that which stands in the Thousands' place—that is, after the fourth Nought. The comma does not stand for any number; it only helps us to see quickly how many? How many Noughts there are. And so we know what kind of number the figures stand for.

10,000, 20,000, 30,000, 40,000, 50,000, 60,000, 70,000, 80,000, 90,000.

What next? Next to Tens of Units we have Hundreds of Units. So next to Tens of Thousands we must have? Hundreds of Thousands. Yes; we call them Hundreds of Thousands, and we give them a new? A new place. Of course we do.

Tens of Thousands have the fifth place, so Hundreds of Thousands must have the? The sixth place. 100,000. 200,000. 300,000. 400,000. 500,000. 600,000. 700,000, 800,000. 900,000.

And what will come next. Next to Hundreds come? Thousands. So next to Hundreds of Thousands come? Thousands of Thousands.

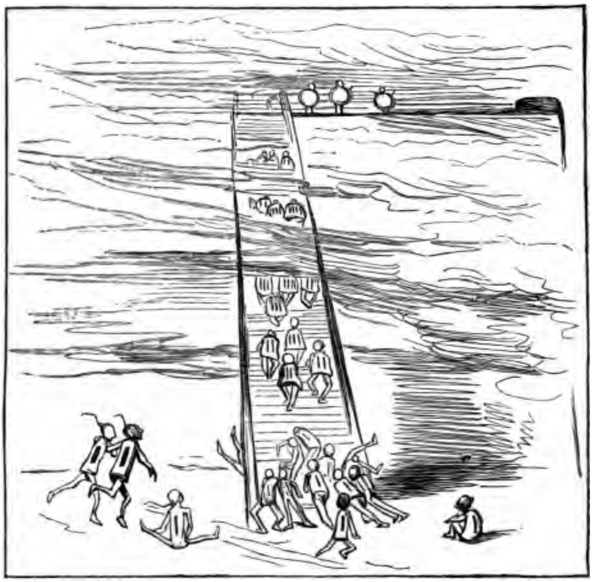
But Thousands of Thousands has another name. The name of a Thousand Thousand is a Million.

And what is the place for a Million? The seventh place. 1,000,000. $1000 \times 1000 = ?$ 1,000,000.

$1 \times 10 = ?$ 10. $10 \times 10 = ?$ 100. $100 \times 10 = ?$ 1000. $1000 \times 10 = ?$ 10,000. $10,000 \times 10 = ?$ 100,000. $100,000 \times 10 = ?$ 1,000,000.

And now we might go on to talk of Tens of Millions and Hundreds of Millions, and we might multiply these by Ten, and so get larger and larger numbers. On and on we might go; need we ever stop? We should have to stop because we should be tired, and because our minds are not big enough to understand such large numbers. But we should not have to stop because Giant Arithmos has not plenty more large numbers, with hard, hard riddles in them. If we were able to think of such very great numbers and really to understand them, I do not see that the time need ever come when the Giant would say,

"Now this number cannot be multiplied by Ten, so you must stop." What is your opinion? Do you not agree with me that there must be no end to the numbers, though it does seem a puzzling thing to think about?



Up they go; need they ever stop?

But people who understand about Millions, and know how to write them, can find out a great many of the Giant's riddles, as you will do, I hope, in time.

EXERCISES.

Write in Arabic figures — One thousand. Three thousand. One thousand five hundred. Two millions. Fifty thousand. Four hundred thousand. One million two hundred thousand. One million two hundred and thirty thousand four hundred and five. Two hundred. Three thousand two hundred. Forty-three thousand two hundred and ten. Five hundred and forty-three thousand two hundred and ten. Six millions five hundred and forty-three thousand two hundred and eleven.

LESSON LVII.

WHAT are the factors of 10? 2 and 5. What do we call a number which has 2 for a factor? An (p. 140) even number.

Then, if one Ten is an even number, all the Tens are? Even numbers also. And 10×10 must be? An even number. So Hundreds are even numbers, and what else? Thousands, and Tens of Thousands, and Hundreds of Thousands, and Millions. Yes, all these are even numbers; they all have Two for a factor.

Suppose we add an even number to another even number, what kind of number will that make? (p. 129). An even number of course; two

even numbers could not possibly make an odd number.

Now suppose Giant Arithmos were to try to puzzle you by showing you a large number, and asking you to tell at once what kind of a number it was, could you do it ?

If it were a Ten, or a Hundred, or a Thousand, or a Ten of Thousands, or a Hundred of Thousands, or a Million, you think you would know directly that it was an even number.

But if the number had in it one Million or some Millions, and some of each of the numbers which are made by multiplying by Ten—Hundreds, and all the rest of them—you would be sure it was an even number, because all these *are* even ; that is, you would be sure, unless there were something else in the number, something which is not a Ten, and is not made by multiplying by Ten.

What is that something ? How did we make the first Ten ? By adding one Unit to another. And the second Ten ? By adding more Units.

When we added an even number to Ten, what kind of number did we make ? An even number.

And when we added an odd number ? We made an odd number.

So if we want to know whether a number is even or odd, we must ask the Units, because we know that the Tens and ? And all the numbers

made by multiplying by Ten are even; so it is the Units which decide whether a number is even or odd.

17,943,218. Is that number even or odd? And this, 289,675,423?

Now tell me, how do we know whether a number is even or odd? By seeing whether the Units are even or odd.

That is a little secret of the Giant's which it is quite worth while to know, because it helps you to find out some of his riddles.

$2 \times 6 = ?$ 12. $2+2+2+2+2+2=?$ 12. So you see that if we knew no Multiplication, and wanted to take a number six times or any number of times, we should be obliged to add it over and over again, till we had added it as many times as we wanted to take it. That would be a very long way of doing what Multiplication does quickly.

$9 + = 9?$ 18. $9+10=?$ 19. When we add Ten, or any number of Tens, what is it we do not change? (p. 162) We do not change the Units when we add Ten or any number of Tens. So we know at once that $9+10=19$.

And we know that as 9 is one Unit less than 10, $9+9$ must = one Unit less than $9+10$.

That shows us that as there must be one Ten more in two Nines than in one Nine, the Units must be One less. 9. 18.

$9 \times 3 = ?$ 27. You see the Units are One

less than in 9×2. 7 is One less than 8. 9. 18. 27.

So every time we take another 9 we make the Tens One more and the Units One less.

If we remember this it make the 9 line very easy. $9 \times 4 = ?$ 36. $9 \times 5 = ?$ 45. $9 \times 6 = ?$ 54. $9 \times 7 = ?$ 63. $9 \times 8 = ?$ 72. $9 \times 9 = ?$ 81. $9 \times 10 = ?$ 90. $9 \times 11 = ?$ 99. $9 \times 12 = ?$ 108.

9
18
27
36
45
54
63
72
81
90
99
108

9, 8, 7, 6, 5, 4, 3, 2, 1, 0, 9, 8. There you see how the Units become One less each time.

Only, as $10 \times 9 = 90$, and Ninety is a Ten, the next Nine does not make another Ten, but begins over again with Nine, for, of course, 11×9 must $= 9$ more than 10×9. Besides, the 0 in the Units' place shows that the number is a Ten, and Nine is One less than Ten. This is another little secret which it is a good thing to know.

When we began these lessons I told you we should want some bricks and some other things to help us. This is our last lesson but one; the next will be the last of all; and we shall want something which we have not had before to help us to understand. What do you think that thing will be, or perhaps I should say those things? I hardly think you will guess, so I had better tell you at once. We shall want an apple, and a plate and knife. Do you ask if we are to eat the apple? We shall see what we shall see, if our apple be quite ripe.

LESSON LVIII.

Now what is to be done with this apple? First tell me how many apples there are? Just one.

Then, as we found long ago (p. 44), we could not both have this one apple. If we both wanted it what should we do? Should we not take the knife and cut the apple in half? Yes, I think that is what we should do, and that is what we *will* do now.

There the apple is cut in half—how many pieces are there? There are two pieces. So there are. When we cut anything in half we cut it into two pieces, not into three or four, but always into two pieces.

So when we speak of half we always mean that something has been divided into how many parts? Into two parts—half means one of these parts.

Now if we wanted to write half in figures, how should we do it? It would not do to put 1. 1 means one whole thing, not one half. We should write 1 for the whole apple, and it will not do to use the same figure for one half as for one whole.

But as we want to write one half, we must have the 1 after all; only it must be written in this way— $\frac{1}{2}$.

The 2 underneath the little line is to show how many parts the thing has been divided into. The 1 shows how many of these parts we wish to speak about or to take.

Let us cut the halves of the apple again; we will cut each half into half. Now how many pieces are there? Four pieces, all of the same size.

What do we call those four pieces? We call them quarters.

Quarters come from the Latin word *Quartus,* which means fourth. And a quarter *is* a fourth. There are four pieces, and a quarter is one of them.

How must we write a quarter? $\frac{1}{2}$ means? One half. So $\frac{1}{4}$ means one quarter.

Cut each piece again into half. How many are there now? Now there are eight pieces. We call each piece an eighth. It is easy to understand what that means, and to know how to write it. Of course it must be written— $\frac{1}{8}$.

Cut each eighth into two equal pieces. Now how many are there? $8 \times 2 = 16$. So there are 16 pieces. What must we call each piece? A

sixteenth? Yes, that is its name; and it must be written thus— $\frac{1}{16}$.

How many halves are there in one thing? Two halves. And how many quarters? Four quarters. How many eighths? Eight eighths. How many sixteenths? Sixteen sixteenths.

Now you can tell, I think, what is the half of 3? The half of 2 is 1; but in 3 there is an odd 1, which must be divided into half. So half of 3 is $1\frac{1}{2}$, One and a-half.

We might go on finding the halves of other odd numbers: you can do so by yourself if you like, but you will have to find out more riddles about whole numbers before you learn much about these broken ones, so that we need not say very much about them to-day. Only if you hear any one speak of quarters, or eighths, or sixteenths, or any other *ths,* you can think that you know what is meant.

The *ths* mean that a thing has been divided, and the beginning of the word tells into how many equal parts the thing has been divided.

Would you rather have $\frac{1}{16}$ or $\frac{1}{8}$ of an apple? If it were a nice apple I think you would rather have $\frac{1}{8}$ than $\frac{1}{16}$, and perhaps you would like $\frac{1}{4}$ still better than $\frac{1}{8}$? Why do I think so? Because when a thing is only divided into four, the pieces are larger than when it is divided into eight or sixteen.

When numbers are used in this way to speak

of parts of anything, and not of whole things, they are called Fractions.

The name Fractions comes from *Fractus,* a Latin word, which means *broken.*

Things can be broken as well as cut, and numbers can be broken too. You know we broke 3 in half, and found that $3 \div 2 = 1\frac{1}{2}$.

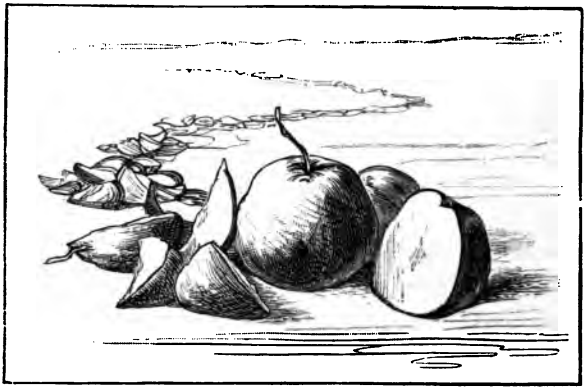
Suppose we had broken 3 into three equal parts, then we should have called each part a third of 3, and written it thus— $\frac{1}{3}$ of 3. $3 \div 3 = 1$, so $\frac{1}{3}$ of $3 = 1$.

We might have cut the apple into three, or into five, or six, or any number of equal parts we liked. Then we should have called each part a third, a fifth, a sixth, or by the name of whatever number of parts we had cut it into.

Let us look once more at our sixteen pieces. Could we cut them again? Yes, we could, only they would be rather small. Still we might manage to divide each of these small pieces, and so we might go on till the pieces got quite too small for our knife. But the tiny bits of apple would be there, and if we had a knife small enough, and hands clever enough, we might go on dividing. Ah, but then we should want eyes sharp enough to see such very little pieces.

But we might put one of the little pieces in a microscope, and then it would look quite large, and there are wonderful little knives made on purpose for cutting tiny things in microscopes.

But at last the little pieces would get too small to be seen even in a microscope; should we then have cut the apple into nothing? Oh no, the little pieces would still be there, only our eyes would not be able to see them, and our minds can hardly think of anything so very very



Cut the Apple.

small, any better than they can think of any numbers so very large as those we talked about the other day (p. 203).

Here are some sums which you can do at once.

1×2. 2×2. 4×2. 8×2. 16×2. 32×2. 64×2. 128×2. 256×2. 512×2. 1024×2.

If we had cut our Sixteenths in half, we should have had to speak of one of the new pieces as $\frac{1}{32}$, one Thirty Second.

If we wished to speak of more than $\frac{1}{32}$, we

might say $\frac{17}{32}$, or $\frac{21}{32}$, or $\frac{31}{32}$, or any number we liked. $\frac{31}{32}$ would be very nearly the whole apple; only one tiny piece, $\frac{1}{32}$, would be left.

If we were to divide the $\frac{32}{32}$, we should have $\frac{64}{64}$. We might go on to cut these $\frac{64}{64}$ into $\frac{128}{128}$, and these again into $\frac{256}{256}$, and so on to $\frac{512}{512}$, and $\frac{1024}{1024}$, and as much further as we chose—that is, if, as I said before, our hands were clever enough.

Now have we cut the apple too small to be eaten, or will you finish this last lesson by eating these Fractions, and how many Fractions will you eat if you please?

CONCLUSION.

So we have finished all our lessons, and you have found out many riddles, none of them very hard, but they will help you, I hope, to find out harder ones, for you know that the harder riddles you can find out, the more Giant Arithmos will be willing to help you.

It is time for you to have another riddle book, but before this is quite finished there is one thing which you may think I ought to tell you, and that is how Giant Arithmos can reach the stars, for you may remember that at the beginning of this book I said that he could do so.

Have you ever been in a train? No doubt you have. Sometimes, perhaps, you went quite a short journey, and sometimes you may have been a long time on the way, as long as from di-

rectly after breakfast till bed-time. Then, when you got out of the train you found yourself at a place far away from where you started.

But did you ever go by train to the sun? Oh no, you say, you *could* not do that? Why not? Because there *is* no train to the sun, and there could not be one.

Well, but *why* could there not be a train to the sun? You think that the rails would have to be put up in the sky, and they would not stay there; they would fall down; and so the train could not go. Yes, that is one reason why there could not be a train to the sun.

I will tell you why, even if there were a train, you could not go to the sun. Suppose that, somehow or other, such a railway had been made, and that you had been put into the train when you were a very little baby, and that the train had started directly and gone as fast as the fastest train ever did go.

How long do you think the journey would have lasted? Would you have got to the sun that day? No, not that day, nor that year. The sun is so very far away that you would not be there now, even if the train kept on going very fast, and you never got out of it. You might grow up in the train, and stay there till you were very old, if you lived so long, and still you would not reach the sun. So, even if there were a train, you would never come to the end of the journey.

No one has ever been to the sun and come back to tell us about it, yet we know how far off it is. It is more than 90,000,000 of miles away.

People even know how much the sun weighs, and many other things about it.

The stars are still farther from us. Yet the same things are known about them. How can this be? It could not be without Giant Arithmos. With his great numbers he can reach the sun and the stars, and help us to find out about them. Do you want to know *how* he does this? Ah, that is one of his very hard riddles—altogether too hard for us to try to understand at present. Perhaps some day, if you go on learning and finding out, you may be able to do something with this very hard riddle, but not just yet.

But we must not end by talking of one thing which you cannot do, when there are so many things you can do. You *can* go on learning more and more Arithmetic; you *can* try to find out things for yourself; and I hope you *can* say that you like Giant Arithmos pretty well, and will be glad to know him better still.

And now I *can* say "Good-bye," because we have come to the end of this little riddle book.

Good-Bye.

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